Exploding Offers and Unraveling in Two-Sided Matching Markets

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August 1, 2016

Abstract

Many two-sided matching markets tend to unravel in time with transactions becoming inefficiently early. In a two-period decentralized model, this paper shows that when a market culture allows firms to make exploding offers, unraveling is more likely to occur and lead to a less socially desirable matching outcome. However, the use of exploding offers is only a necessary but not a sufficient condition for unraveling to occur. A market with an excess supply of labor is less vulnerable to the presence of exploding offers; yet the conclusion is ambiguous for a market with a larger uncertainty in early stages, which depends on the specific information structure. While a banning policy on exploding offers tends to be supported by high quality firms and workers, it can be opposed by those of low qualities. This explains the prevalence of exploding offers in practice.

1 Introduction

Many two-sided matching markets exhibit the tendency to unravel in time with transactions occurring earlier and earlier. In these markets, with participants' qualities gradually revealed over time, early transactions can lead to significant efficiency losses: due to the lack of information in early stages, a higher probability of mismatch often leads to instability and thus costly rematching procedures afterwards.[1] In order to effectively halt such an unraveling process, the previous literature has identified several factors that may influence early moving incentives; one of them is the use of exploding offers.

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[1]Roth and Xing (1994) provide a detailed overview of various evidence for market unraveling. See also Mongell and Roth (1991), Haruvy, Roth, and Ünver (2006), Avery, Fairbanks, and Zeckhauser (2009), Avery et al. (2001), and Fréchette, Roth, and Ünver (2007).

An exploding offer is an offer that comes with a time limit. The offer has to be accepted within the time limit, or it is considered rejected. In contrast, an open offer can be held until the end of the market. According to Niederle and Roth (2009), exploding offers are prevalently used in many markets facing serious unraveling problems, such as the market for new gastroenterologists in the US. In a lab experiment, they are able to reproduce the facilitating effect of exploding offers on early transactions. However, not all markets that use exploding offers suffer from unraveling. For example, in the job market for junior economists, offers often come with short time limits, yet little tendency towards unraveling has been observed so far.

This paper aims to reconcile the above phenomena from a theoretical perspective. In a two-period decentralized matching market, the true qualities of workers are not fully revealed until the second period; firms and workers only observe a signal in the first period. By comparing markets with or without a banning policy on exploding offers, I show that when firms are allowed to make exploding offers, equilibria without unraveling are supported by a smaller parameter space and thus a stable matching is less likely to be achieved. Intuitively, a worker tends to exploit an open offer by holding it until the last period. In response, a firm uses an exploding offer to eliminate the risk of being rejected and remaining unmatched at end of the market. The time limit forces the worker to balance the cost of rejecting the current offer and the likelihood of receiving a better offer in the future. As a result, an early exploding offer is more easily accepted than an early open offer, and thus the market is more likely to unravel when exploding offers are allowed. In addition, I identify the sufficient conditions under which an equilibrium without unraveling never exists, and the sufficient and necessary condition for the market to fully unravel.

The above results indicate that the use of exploding offers is only a necessary but not a sufficient condition for unraveling to occur.[2] An important question we should ask is what characteristics of a market can make it less likely to be affected by exploding offers? This study highlights two findings on comparative statics. First, a market tends to be less vulnerable when there is an excess supply of labor, that is, when workers outnumber job vacancies in the market. In this case, low quality workers remain unmatched in an equilibrium without unraveling, and a firm incurs the risk of hiring these workers by moving before the resolution of uncertainty. Therefore, the firm may be unwilling to deviate even when an early exploding offer would be accepted. Similar intuitions are also present in Niederle, Roth, and Ünver (2013), in which they find unraveling tends to occur in a market with comparable demand and supply.[3]

Second, signal accuracy has an ambiguous effect on how vulnerable a market is to exploding

[2]In a different environment, Fainmesser (2013) also shows that the use of exploding offers is a necessary condition for unraveling to occur.

[3]On a related note, Niederle and Roth (2003) suggest that the unraveling in the gastroenterology market might have been triggered by a demand shock. McKinney, Niederle, and Roth (2003) conduct an experimental research on demand and supply imbalances tailored to this market.

offers; the conclusion depends on the specific information structure. As mentioned above, in the first period, market participants only observe a signal suggesting each worker's potential quality or type. As the signal becomes more accurate, a firm is more willing to make an early exploding offer to a high type worker while the worker is less willing to accept; a firm is less willing to make an early exploding offer to a low type worker while the worker is more willing to accept. In other words, the early moving incentives always change in opposite directions for different sides of the market. The overall effect of signal accuracy hinges on how uncertainty is resolved in a specific market. Under the same framework, I show that opposite effects can be produced by simply altering the information structure of the market.

Such a finding reconciles the different conclusions from two previous studies. Roth and Xing (1994) find that unraveling tends to be impeded if the uncertainty in early stages is sufficiently large, while a non-monotonic relationship is shown in Fainmesser (2013): an increase in signal accuracy generates greater market unraveling when the signal is inaccurate enough, while the effect reverses for a market with an accurate signal. This paper suggests that the information structure is a key element that can affect the comparative statics on signal accuracy.

Given the facilitating effect of exploding offers on early transactions, a natural policy consideration is whether exploding offers should be allowed in a market. I investigate the welfare aspect by asking the following questions. Suppose there is a proposal on a ban towards exploding offers. Who would support? Who would oppose? The result indicates that such a banning policy can be supported by high quality firms and workers but opposed by agents of lower qualities. This provides a theoretical support for the experimental findings in Niederle and Roth (2009). In their exploding offer treatment, some higher (lower) quality firms and workers receive significantly lower (higher) payoffs than in the open offer treatment. The conclusion also explains the prevalence of exploding offers in practice and why it is sometimes difficult to achieve consensus on how to solve the unraveling problem in real markets.[4]

The basic model is then extended into three periods. A comparison between exploding offers of different time limits indicates a continuation in the effects: in a market with exploding offers of shorter time limits, the equilibrium without unraveling is less likely to sustain. Moreover, by allowing concavity (convexity) in quality distributions, the extended model shows that a market is less likely to be affected by exploding offers if the quality distribution over firms is more convex, or that over workers is more concave.

Besides the use of exploding offers and the market characteristics discussed above, the previous literature has identified several other factors that could affect unraveling, including the stability of centralized matching algorithms (Roth, 1984, 1991; Kagel and Roth, 2000; Ünver, 2001, 2005), market congestion (Roth and Xing, 1997), the quality distribution over participants on each side (Niederle, Roth, and Ünver, 2013), similarity of preferences

[4]For example, see a discussion regarding the US market for new gastroenterologists by Niederle and Roth (2005).

(Hałaburda, 2010), social network structures (Fainmesser, 2013), and strategic complementarities (Echenique and Pereyra, 2016). The motives behind unraveling are also investigated in some different environments. Under the framework of competitive markets, Li and Rosen (1998), Li and Suen (2000, 2004), and Suen (2000) show how unraveling can occur as a form of insurance in the absence of complete markets. In a model with asymmetric information, but without evolving uncertainty, Lee (2009) explains early contacting as a way to avoid adverse selection.

The subsequent analysis proceeds as follows. Section 2 describes the model. Section 3 provides the equilibrium predictions. In Section 4, I give a simple welfare analysis and discuss some policy issues. Some extensions are examined in Section 5, and Section 6 concludes.

2 The Model

Consider a two-sided matching market with F firms and W workers, where $F \geq 3$ and $W \geq 3$. Let $\mathcal{F} = \{f_1, ..., f_F\}$ be the set of firms, and $\mathcal{W} = \{w_1, ..., w_W\}$ be the set of workers. Each firm has the capacity to hire at most one worker, and each worker can work for at most one firm. A market with $W < F$ (or $W > F$) is said to have *excess demand* (or *excess supply*) of labor.

All workers agree on the same ranking of firms: $f_F \succ f_{F-1} \succ ... \succ f_1$, and all firms agree on the same ranking of workers: $w_W \succ w_{W-1} \succ ... \succ w_1$.[5] The ranking of firms is common knowledge to the entire market. The true ranking of workers is revealed over time. Let $\mathcal{R}$ be the set of all possible strict rankings of workers, in which each ranking/state is realized with equal probability $\frac{1}{W!}$. Denote the true ranking/state as $\succ \in \mathcal{R}$.

In terms of utility, all firms value a match with the i-th ranked worker in the true state (w_i) as $v_i = i$, and all workers value a match with the j-th ranked firm (f_j) as $u_j = j$. Unmatched market participants derive zero utility: $v_0 = u_0 = 0$.[6] Therefore, any match is preferable to remaining unmatched. Notice in this setting, a firm's utility from a match depends only on the worker's rank, and a worker's utility depends only on the firm's rank. The two functions v_i and u_j indicate worker quality and firm quality respectively.[7]

The outcome of a matching market, that is, a matching is said to be *stable* if and only if there is no worker–firm pair in which each prefers one another to her current match. Since the existence of such pairs often leads to costly rematching procedures afterwards, stability is used as a central criterion to evaluate market outcomes by the two-sided matching literature.

[5]Hałaburda (2010) considers the similarity of firms' preferences over workers as a comparative statics parameter while having all workers agree on the same ranking of firms. The result shows that similarity of preferences is an important factor driving unravelling.

[6]Normalizing the utility range to be between 0 and 1 would not change the main results of the paper.

[7]An example that allows non-linear utility distributions can be found in Section 5. On a related note, Niederle, Roth, and Ünver (2013) show in a lab experiment that unraveling only occurs when demand and supply are comparable, that is, when there exist excess workers, but a shortage of high quality workers.

In the current environment with strict rankings and aligned preferences, it is easy to see that the assortative matching in the true state $\succ$ constitutes the unique stable matching.

The market lasts for two hiring periods, with the true ranking $\succ$ revealed in Period 2. At the beginning of Period 1, a public signal $\hat{\succ} \in \mathcal{R}$ is observed by both firms and workers. With probability α, $\hat{\succ}$ is the same as $\succ$. Otherwise, $\hat{\succ}$ is a uniform random draw from $\mathcal{R}$. The parameter $\alpha \in (0, 1)$ measures signal accuracy: a larger value of α indicates a smaller uncertainty faced by the market. Denote a worker's type in Period 1 as $\hat{r}$, which is her rank in $\hat{\succ}$. Denote a worker's true rank as r, which is her rank in $\succ$.

The game proceeds as follows. In Period 1, a public signal $\hat{\succ}$ is observed. Next, each firm simultaneously makes an offer to at most one worker. Finally, each worker simultaneously chooses at most one offer to accept from those available to her. A similar procedure takes place in the second period, except that the true ranking of workers $\succ$, instead of a signal, is observed at the beginning of the period. All actions of firms and workers are publicly observed.

I focus my discussion on two types of offers: exploding offers and open offers.

Definition 1. An *exploding offer* is an offer that comes with a time limit. It can only be accepted within the time limit. Otherwise, it is rejected.

Definition 2. An *open offer* is an offer that can be held until the last period.

In the current two-period model, an exploding offer has to be accepted immediately, in the same period in which it is made. However, if a worker receives an open offer in the first period, she could choose to hold it until Period 2. An open offer made in Period 2 is equivalent to an exploding offer.

Regarding the culture or norms of the market, I make the following two assumptions.

Assumption 1. (Binding acceptances) Once a worker accepts an offer, the acceptance is binding. A worker cannot renege on her acceptance.

Assumption 2. (Binding rejections) Once a worker rejects an offer, the rejection is binding. A firm will not make an offer to the same worker again.

Following Niederle and Roth (2009), Assumption 1 is made to ensure the validity of exploding offers. Assumption 2 is an important and reasonable addition because (i) it reflects the norms of some real-life two-sided matching markets such as the market for judicial clerks; (ii) it increases the power of exploding offers by raising workers' rejection costs. Hence, without such an assumption, the effects of exploding offers on market outcomes can be largely underestimated.[8]

[8] The reality in some markets is less stringent than Assumption 2. For example, in the job market for junior economists, although when rejecting an exploding offer, a candidate typically does not consider the possibility

3 Equilibrium Analysis

In this section, I start with a baseline case where firms can only make open offers, and then relax the constraint by allowing exploding offers. The discussion mainly concerns two types of subgame perfect Nash equilibria in weakly undominated pure strategies: those without unraveling, and those with full unraveling.

The following characterization of equilibria focuses on the timing of offers made by "relevant" firms, which include all firms as $W \geq F$ and only firms f_F, f_{F-1},..., and f_{F-W+1} as $W < F$. This is because in either type of equilibria, f_{F-W}, f_{F-W-1}, ..., and f_1 as $W < F$ are indifferent among all possible strategies and their actions do not affect the equilibrium outcome.

Definition 3. An *equilibrium without unraveling* is an equilibrium where no relevant firms make any offers until the last period.

In an equilibrium without unraveling, no actions are taken by relevant firms in Period 1. In Period 2, two cases are considered separately: (i) when $W \geq F$, f_F makes an offer to w_W, f_{F-1} to w_{W-1}, ..., f_1 to w_{W-F+1}, and all offers are accepted; (ii) when $W < F$, f_F makes an offer to w_W, f_{F-1} to w_{W-1}, ..., and f_{F-W+1} to w_1, and all these offers are accepted.

Definition 4. An *equilibrium with full unraveling* is an equilibrium where every relevant firm makes an early offer in Period 1.

In an equilibrium with full unraveling, after $\hat{\succ}$ is revealed in Period 1, two cases are considered separately: (i) when $W \geq F$, f_F makes an offer to type $\hat{r} = W$, f_{F-1} to $\hat{r} = W-1$, ..., f_1 to $\hat{r} = W - F + 1$, and all offers are accepted; (ii) when $W < F$, f_F makes an offer to type $\hat{r} = W$, f_{F-1} to $\hat{r} = W - 1$, ..., f_{F-W+1} to $\hat{r} = 1$, and all these offers are accepted.

In both types of equilibria, when $W < F$, firms f_{F-W}, f_{F-W-1}, ..., and f_1 may adopt any strategy; their offers (if made) are not accepted by any workers. Clearly, the unique outcome of equilibria without unraveling is the assortative matching according to the true ranking of workers $\succ$, which is the unique stable matching in the current setting. The unique outcome of equilibria with full unraveling is the assortative matching according to the signal-suggested ranking of workers $\hat{\succ}$, which is only stable when the signal is correct.

In addition, an equilibrium is said to have *partial unraveling* if some relevant firms make offers in Period 1, and some make offers in Period 2.

that she may receive an offer from the same employer again, the phenomenon of nonbinding rejections is still observed in some situations. In this case, a more realistic setting is to have each firm decide whether to raise its leverage by attaching a commitment of binding rejection when making an early exploding offer. For these markets, although Assumption 2 significantly simplifies the analysis, it can lead to an overestimation of the effects of exploding offers.

3.1 Open Offers Only

Consider the case where firms can only make open offers due to the culture, norms, or policies in a market environment.[9] The following proposition describes the equilibrium outcome.

Proposition 1. *When firms are not allowed to make exploding offers, there only exist equilibria without unraveling; the stable matching is the unique equilibrium outcome.*

While the full proof is provided in the appendix, the basic intuition is clear. In Period 1, every type of worker has a positive probability of having the highest quality in the true state. Therefore, as long as the best firm f_F moves in Period 2, a worker strictly prefers to hold any early open offer until Period 2. Knowing this, f_F strictly prefers to wait until the last period, so that all workers will stay in the market and the one of the highest quality can be perfectly identified. Since no offer is accepted in Period 1, the other firms cannot make themselves better off by moving early; instead, they incur the risk of being rejected in the last period and remaining unmatched.

Proposition 1 shows that in a market where firms only use open offers, an equilibrium without unraveling always exists, while an equilibrium with full unraveling never does. This provides us with a very clean baseline, so that the effects of exploding offers can be easily identified from the change in the parameter spaces supporting these two types of equilibria.

3.2 Exploding and Open Offers

Now I consider the case where both open offers and exploding offers can be made in a market.

Lemma 1. *In an equilibrium in undominated strategies, firms f_{F-1}, f_{F-2}, ..., and f_1 never make an open offer in Period 1 when they are allowed to make exploding offers.*

The best firm f_F is indifferent between an open offer and an exploding offer since neither of them will be rejected by any worker. In Period 2, every firm is indifferent because an open offer is equivalent to an exploding offer. Hence, in the subsequent analysis of equilibria in weakly undominated strategies, Lemma 1 allows us to consider only exploding offers without loss of generality.

After observing a signal $\hat{\succ}$in Period 1, both firms and workers update their beliefs. Posteriors on the true state $\succ$ are given by

$$\Pr(\hat{\succ} \mid \hat{\succ}) = \alpha + \frac{1-\alpha}{W!},$$

[9] For example, exploding offers are publicly discouraged in the US market for new graduate students. The Council of Graduate Schools has published a resolution stating that students are under no obligation to respond to offers of financial support prior to April 15 (http://cgsnet.org/april-15-resolution).

and

$$\Pr(\succ' | \hat{\succ}) = \frac{1-\alpha}{W!}, \; \forall \succ' \neq \hat{\succ}.$$

Posteriors on the true rank r of a type-$\hat{r}$ worker are given by

$$\Pr(\hat{r} \mid \hat{r}) = \alpha + \frac{1-\alpha}{W},$$

and

$$\Pr(r' \mid \hat{r}) = \frac{1-\alpha}{W}, \; \forall r' \neq \hat{r}.$$

The posteriors described above involve a spike at the point $\hat{\succ}$, which indicates a high probability of the true state being the signal-suggested ranking, and an equally low probability of any other ranking being realized.[10] Therefore, the assortative pairs in the state $\hat{\succ}$ will be frequently used in the subsequent analysis. These "signal-suggested pairs" are formally defined as follows.

Definition 5. The *signal-suggested type* of a firm f_j is a function defined as

$$\hat{r}(j) \equiv \begin{cases} j + W - F & \text{if } j > F - W, \\ 0 & \text{if } j \leq F - W. \end{cases}$$

The *signal-suggested firm* of a type $\hat{r}$ is a function defined as

$$j(\hat{r}) \equiv \begin{cases} \hat{r} - W + F & \text{if } \hat{r} > W - F, \\ 0 & \text{if } \hat{r} \leq W - F. \end{cases}$$

A firm and its signal-suggested type, or a type of worker and its signal-suggested firm, are called a *signal-suggested pair*.

3.2.1 Equilibria without Unraveling

As mentioned in the introduction, in practice not all markets that use exploding offers suffer from unraveling. Proposition 2 provides sufficient conditions for an equilibrium without unraveling to sustain even when exploding offers are allowed in a market, while Proposition 3 provides sufficient conditions for such an equilibrium never to exist.

Recall a market with $W < F$ (or $W > F$) is said to have excess demand (or excess supply) of labor. I further identify the case of extreme excess supply if $W \geq 2F$, and the case of moderate excess supply if $F < W \leq 2F - 1$.

[10] In Section 3.3, I provide an example of a different information structure, under which the probability of a ranking gradually decreases as its Kendall τ distance to the signal-suggested ranking increases.

Proposition 2. *When firms are allowed to make exploding offers, an equilibrium without unraveling always exists if*

(i) the market has an extreme excess supply of labor; or

(ii) the market has a moderate excess supply of labor and a sufficiently inaccurate signal.

Mathematically speaking, an equilibrium without unraveling always exists if (i) $W \geq 2F$, or (ii) $F + 2 < W \leq 2F - 1$ and $\alpha \leq \frac{(W-F)^2 - W + F - 2}{(W-F)^2 + W + F - 2}$. A deviation from such an equilibrium involves both sides of the market: a firm should want to make an early exploding offer in Period 1 to a worker who wants to accept it. Hence, the condition for an equilibrium without unraveling to sustain requires that, for each worker type $\hat{r}$, the firms whose offers would be accepted should not be willing to offer. In most cases, the signal-suggested firm-worker pairs have the strongest incentive to deviate, thus affecting the binding constraint that drives the results in Proposition 2. Below I use an example to explain why.

Example 1. *In a market with 3 firms and 3 workers, an equilibrium without unraveling yields the following assortative matching:*

$$\begin{matrix} w_3 & w_2 & w_1 \\ f_3 & f_2 & f_1 \end{matrix}.$$

Consider a deviation between f_2 and a worker of type $\hat{r} = 3$. Given the posteriors on this type in Period 1, the offer will be accepted if $2 \geq \left(\alpha + \frac{1-\alpha}{3}\right) \times 3 + \frac{1-\alpha}{3} \times 1 + \frac{1-\alpha}{3} \times 1$. While the LHS gives the worker's utility when accepting the offer, the RHS is the worker's expected utility when rejecting. The second part of the RHS indicates a rejection cost of $\frac{1-\alpha}{3}$: after rejecting f_2, the worker can only receive an offer from f_1 even when her true rank turns out to be $r = 2$ in Period 2. On the other hand, if f_2 makes an early offer to its signal-suggested type $\hat{r} = 2$, the offer will be accepted when $2 \geq \frac{1-\alpha}{3} \times 3 + \left(\alpha + \frac{1-\alpha}{3}\right) \times 1 + \frac{1-\alpha}{3} \times 1$. Again, the second part of the RHS indicates a rejection cost of $\left(\alpha + \frac{1-\alpha}{3}\right)$.

Example 1 shows that a deviation between a signal-suggested pair tends to succeed more easily: although a firm always prefers higher types, its signal-suggested type is more likely to accept its early offer due to the higher rejection cost ($\alpha + \frac{1-\alpha}{3} > \frac{1-\alpha}{3}$). This is quite intuitive since it indicates that a worker is more reluctant to reject what appears to be "a good match" in Period 1 — a firm that is most likely to be her match in a stable matching. Therefore, for an equilibrium without unraveling to sustain, the binding condition in most cases requires the best firm that would be accepted by its corresponding type not be willing to offer.

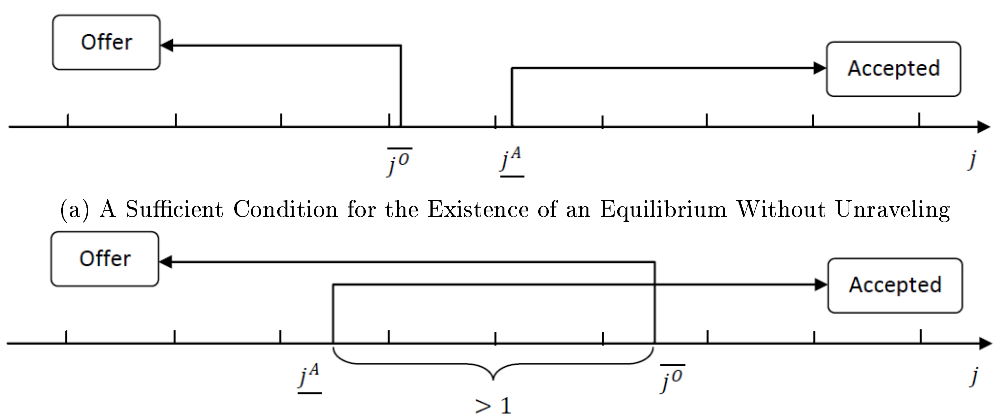
Condition (ii) of Proposition 2 is mainly driven by the workers' side of the market. As the signal in Period 1 becomes less accurate, the cost of rejecting a signal-suggested firm $\left(\alpha + \frac{1-\alpha}{3}\right)$, or more generally, $\left(\alpha + \frac{1-\alpha}{W}\right)$ decreases. Thus, a worker is more likely to reject an exploding offer and an equilibrium without unraveling is more likely to sustain.[11] On the other hand, condition (i) stems from a boundary solution on the firms' side, in which case no firm is willing to make an early offer to its signal-suggested type even if it is always accepted. It arises when there is an extreme excess supply of labor ($W \geq 2F$), which means in an equilibrium without unraveling, even the worst firm f_1 is matched with an above-average worker. This significantly increases a firm's risk in making an offer before the resolution of uncertainty since the worker is more likely to have a lower quality compared to the firm's match in equilibrium.

Proposition 3. *When firms are allowed to make exploding offers, an equilibrium without unraveling never exists if*

(i) the market has an excess demand of labor and a sufficiently accurate signal; or

(ii) the market has a moderate excess supply of labor and a sufficiently accurate signal.

Mathematically speaking, an equilibrium without unraveling never exists if (i) $W \leq F$ and $\alpha > \frac{W-1}{2W-1}$, or (ii) $F < W < 2F-1$ and $\alpha > \frac{(W-F)^2+W+F-2}{(W-F)^2+3W+F-2}$. For a deviation to occur, there must exist a type of worker and a firm in Period 1 such that, the firm is willing to offer and the worker is willing to accept. Again, the binding condition here hinges on the deviations between signal-suggested pairs. Proposition 3 shares the similar intuition with Proposition 2: a more accurate signal encourages a deviation by increasing the worker's rejection cost, while an excess demand of labor decreases the firm's risk of being worse off in a deviation.



(a) A Sufficient Condition for the Existence of an Equilibrium Without Unraveling

(b) A Sufficient Condition for the Non-existence of an Equilibrium Without Unraveling

Figure 1: An Integer Problem

[11] Although the cost of rejecting a non-signal-suggested firm $\frac{1-\alpha}{W}$ increases as the signal becomes less accurate, here the elimination of such a deviation is not the binding condition and does not drive the results. In Section 3.3, I provide an example with a different information structure, under which the non-signal-suggested pairs can also influence the binding condition, and thus signal accuracy may have the opposite effect on market unraveling.

There exists a gap between the two parameter spaces identified in Propositions 2 and 3. Let α_1 and α_2 be functions of W and F such that $\alpha_1(W,F) \equiv \frac{(W-F)^2-W+F-2}{(W-F)^2+W+F-2}$ and $\alpha_2(W,F) \equiv \frac{(W-F)^2+W+F-2}{(W-F)^2+3W+F-2}$. For some cases of moderate excess supply ($F+2 < W < 2F-1$), an equilibrium without unraveling always exists when $\alpha \leq \alpha_1$ and never exists when $\alpha > \alpha_2$. It is easy to confirm that $\alpha_1 < \alpha_2$.

Such a gap is due to an integer problem, which makes conditions in Propositions 2 and 3 sufficient but not necessary. Denote $\underline{j^A}$ as the lowest quality firm whose early offer is accepted by its signal-suggested type and $\overline{j^O}$ as the highest quality firm willing to make such an offer given the acceptance. For an equilibrium without unraveling to exist, a sufficient and necessary condition only requires no integer to be in the range between $\underline{j^A}$ and $\overline{j^O}$, which depends on the specific parameter values of α, W, and F. In order to draw a general conclusion, we need to have the range completely empty, that is, $\underline{j^A} \geq \overline{j^O}$ (Figure 1a). Similarly, although an equilibrium without unraveling does not exist as long as there is an integer in the range between $\underline{j^A}$ and $\overline{j^O}$, for a general conclusion, we need to set the range larger than 1, that is, $\overline{j^O} - \underline{j^A} > 1$ (Figure 1b).

For cases within the gap, whether unraveling will occur depends on the specific values of α, F, and W. To illustrate, I provide the following examples where $F+2 < W < 2F-1$ and $\alpha \in (\alpha_1, \alpha_2]$. An equilibrium without unraveling exists in case (1) but not in case (2).

Example 2. (1) *When $\alpha = 0.51$, $F = 5$, and $W = 8$, there does not exist an equilibrium without unraveling: given $\underline{j^A} \approx 0.7$ and $\overline{j^O} = 1.5$, we know that the signal-suggested pair f_1 and $\hat{r} = 4$ has an incentive to deviate.*

(2) *When $\alpha = 0.51$, $F = 7$, and $W = 12$, an equilibrium without unraveling can sustain since there is no integer between $\underline{j^A} \approx 1.2$ and $\overline{j^O} = 1.5$.*

3.2.2 Equilibria with Full Unraveling

Now we discuss an extreme case, where the market fully unravels with every relevant firm making an exploding offer in Period 1.[12]

Proposition 4. *When firms are allowed to make exploding offers, an equilibrium with full unraveling exists if and only if $W \leq F$ and $\alpha \geq \frac{W-2}{W}$, that is, the market has an excess demand and a sufficiently accurate signal.*

When $W > F$, an equilibrium with full unraveling can never sustain because after Period 1, there are $W - F$ workers left in the market. Then every firm has an incentive to deviate by waiting, in which case it becomes the only available firm in Period 2 and its choice set is expanded by $W - F$ workers after the resolution of uncertainty. When $W \leq F$, however, a firm has no incentive to wait as long as in its deviation, no worker would reject an offer from

[12] Recall that relevant firms include all firms as $W \geq F$, and only firms f_F, f_{F-1},..., and f_{F-W+1} as $W < F$.

her signal-suggested firm and become available in Period 2. With a more accurate signal, it is more costly for a worker to reject her offer in equilibrium, and thus a deviation is less likely to occur.

3.2.3 Equilibria with Partial Unraveling

In an equilibrium with partial unraveling, only some of the relevant firms make offers in Period 1, while the others choose to wait until Period 2. In contrast to the previous two types of equilibria, partial unraveling may take various forms. In this section, I first rule out some impossible structures of such equilibria, and then demonstrate some possibilities in examples.

Remark 1. There cannot exist an equilibrium with partial unraveling where every firm that moves early has a higher quality than all the firms that choose to wait.

For example, with three firms in a market, there cannot exist an equilibrium with f_3, or f_3 and f_2 being the only early moving firm(s). Such a structure cannot sustain because there exists a profitable deviation for every early moving firm in equilibrium: by deviating to Period 2, it becomes the highest quality firm with an expanded choice set after the resolution of uncertainty.

Combining Propositions 3 and 4, we can identify two parameter spaces where neither an equilibrium without unraveling nor an equilibrium with full unraveling exists: (i) $F < W < 2F - 1$ and $\alpha > \frac{(W-F)^2+W+F-2}{(W-F)^2+3W+F-2}$; (ii) $4 \leq W \leq F$ and $\frac{W-1}{2W-1} < \alpha < \frac{W-2}{W}$. Only equilibria with partial unraveling may exist in these cases. However, as shown in the following example, for some values of W, F, and α, there does not exist any equilibrium.

Example 3. *Suppose there are 4 workers and 3 firms in the market.*

(1) *When $\alpha \in (\frac{3}{7}, \frac{5}{11}]$, there only exists the following equilibrium: in Period 1, f_2 makes an exploding offer to type $\hat{\imath} = 4$ and f_1 makes an exploding offer to type $\hat{\imath} = 2$; in Period 2, f_3 makes an offer to the best worker left in the market; all offers are accepted.*

(2) *When $\alpha \in (\frac{5}{11}, \frac{1}{2})$, there does not exist any equilibrium.*

(3) *When $\alpha \geq \frac{1}{2}$, there only exist the following two equilibria. (i) In Period 1, f_1 makes an exploding offer to type $\hat{\imath} = 2$; in Period 2, f_3 makes an offer to the best worker left in the market and f_2 makes an offer to the second best; all offers are accepted. (ii) In Period 1, f_2 makes an exploding offer to type $\hat{\imath} = 3$ and f_1 makes an exploding offer to type $\hat{\imath} = 2$; in Period 2, f_3 makes an offer to the best worker left in the market; all offers are accepted.*

The above example provides us with some interesting intuitions. In case (1), the highest type worker $\hat{\imath} = 4$ in Period 1 is "stolen" by an early moving firm f_2. But since the signal is not accurate enough, the best firm f_3 still prefers to wait for the resolution of uncertainty. In case (2), as the signal becomes more accurate, $\hat{\imath} = 4$ becomes more attractive but is still willing to accept an early offer from f_2. As a result, f_3 is forced to move early as well so as to prevent such an early transaction; the equilibrium in case (1) can no longer sustain. In case (3) with an even more accurate signal, $\hat{i} = 4$ is no longer willing to accept f_2 in Period 1; both the highest type worker and the highest quality firm choose to wait. Clearly, in the analysis of equilibria with partial unraveling, signal accuracy plays an important role in determining how or whether the early moving incentives for one firm are affected by early offers of other firms, because it tells the high quality firms whether it is worthwhile to "fight" for workers of high types.

3.3 The Ambiguous Effect of Signal Accuracy

According to the above analysis, exploding offers has a facilitating effect on early transactions; such an effect tends to be less salient in a market with an excess supply of labor or a less accurate signal. In this section, I provide an example to show that signal accuracy may have an opposite influence under a different information structure.

Example 4. *There are three workers in the market. Recall $\mathcal{R}$ is the set of all possible strict rankings of workers. The Kendall τ distance, denoted as $K(\succ', \succ'')$, is a function that counts the number of pairwise disagreements between two rankings $\succ'$ and $\succ''$, $\forall \succ', \succ'' \in \mathcal{R}$. In other words, $K(\succ', \succ'')$ measures the distance between $\succ'$ and $\succ''$.*[13] *For instance, between two rankings of three workers, $K(w_1 \succ w_2 \succ w_3, w_1 \succ w_3 \succ w_2) = 1$, $K(w_1 \succ w_2 \succ w_3, w_3 \succ w_1 \succ w_2) = 2$, while $K(w_1 \succ w_2 \succ w_3, w_3 \succ w_2 \succ w_1) = 3$.*

The signal $\hat{\succ}$ in Period 1 equals a ranking $\succ'$ with probability $\frac{1}{6}\left[1 + 3\beta - 2\beta K(\succ', \succ)\right], \forall \succ' \in \mathcal{R}$, which is decreasing in the distance between $\succ'$ and the true state $\succ$. The parameter $\beta \in (0, \frac{1}{3})$ measures signal accuracy: a larger value of β indicates a smaller uncertainty faced by the market. The posteriors are thus given by

$$\Pr(\succ = \succ' | \hat{\succ}) = \frac{1}{6}\left[1 + 3\beta - 2\beta K(\succ', \hat{\succ})\right] \ \forall \succ' \in \mathcal{R}.$$

That is, the probability of a ranking being the true state gradually decreases as its distance to the signal $\hat{\succ}$ increases. This is different from the single spike at $\hat{\succ}$ in the posteriors described in Section 3.2.

Consider the simple case with three firms. An equilibrium without unraveling exists if and only if $\beta \geq \frac{1}{4}$. This result stems from the binding condition that prevents a worker of type $\hat{i} = 3$ from accepting f_2 in Period 1, with f_2 always willing to make such an exploding offer if accepted. In contrast to Proposition 2, here the condition puts a lower bound to signal accuracy, that is, it requires the signal to be sufficiently accurate for the market not to unravel.

The intuition lies in the differences between the two information structures. The posteriors described in Section 3.2 put a high probability on the signal-suggested ranking, and an equally

[13] See Kendall (1938) and Kemeny (1959).

low probability on any other ranking. As a result, the binding condition for no unraveling mainly concerns the deviations between signal-suggested pairs. In such a pair, a more accurate signal increases the worker's rejection cost without affecting the firm's early moving incentive (given the worker's acceptance of its early exploding offer).[14] Overall, a more accurate signal makes the market more vulnerable to the use of exploding offers. Under the information structure in Example 4, however, non-signal-suggested pairs can also influence the binding condition. For these pairs, as the signal becomes more accurate, a high type worker in Period 1 becomes more attractive; firms are more willing to make an early exploding offer while the worker is less willing to accept. Meanwhile, a low type worker becomes less attractive; firms are less willing to offer while the worker is more willing to accept. In other words, the early moving incentives always change in opposite directions for different sides of the market. Then in certain cases such as Example 4, when the binding condition prevents a high type worker from accepting a non-signal-suggested firm, the existence of an equilibrium without unraveling requires the signal to be sufficiently accurate. In contrast to the predictions in Section 3.2.1, here a more accurate signal can make a market less likely to be affected by exploding offers.

The above analysis suggests that the effect of signal accuracy may hinge on how uncertainty is resolved in a specific market, which is different from the prediction of Roth and Xing (1994). Like the basic model of this paper, their model indicates a definitive effect: unraveling tends to be impeded if the uncertainty in early stages is sufficiently large, that is, if the signal is inaccurate enough. However, Example 4 provides an information structure under which unraveling can be facilitated by a sufficiently inaccurate signal.

4 Welfare Analysis

Knowing that the use of exploding offers tends to facilitate unraveling and that unraveling hurts the stability of two-sided matching markets, a natural policy consideration is whether a market should allow firms to make exploding offers. In this part, I investigate the welfare aspect by asking the following questions. Suppose there is a proposal on a ban towards exploding offers. Who would support? Who would oppose? In Section 3, I have identified two types of equilibria that are of the most interest in this environment: those without unraveling, and those with full unraveling. A comparison between these two types of equilibria can shed some light on these questions.

Suppose W, F, and α are such that $4 \leq W \leq F$ and $\alpha \geq \frac{W-2}{W}$. According to previous results, when exploding offers are banned from the market, the market does not unravel (Proposition 1); when exploding offers are allowed, the market always unravels (Proposition

[14]See equations (14) and (16) in the appendix for the expressions of $\underline{j^A}$ (the lowest quality firm whose early offer is accepted by its signal-suggested type) and $\overline{j^O}$ (the highest quality firm willing to make such an offer given the acceptance) respectively. Signal accuracy α only enters the former but not the latter.

3) and an equilibrium with full unraveling exists (Proposition 4).

Comparing to the assortative matching, f_j is better off in a fully unraveled market if

$$j + W - F < \alpha (j + W - F) + \frac{(1-\alpha)(W+1)}{2}, \tag{1}$$

where $j = F - W + 1, F - W + 2, ..., F$.[15] That is,

$$W - F + 1 \leq j < \hat{j} \equiv \frac{2F - W + 1}{2}. \tag{2}$$

From $W \leq F$, we have

$$\hat{j} \geq \frac{F+1}{2}. \tag{3}$$

The LHS of (1) is f_j's payoff in an equilibrium without unraveling, since it is matched with worker w_{j+W-F}. In an equilibrium with full unraveling, f_j is matched with its signal-suggested type $\hat{r}(j) = j + W - F$. The RHS gives the expected payoff. The function $\hat{j}$ is defined as the cutoff firm.[16] Firms ranked lower than $\hat{j}$ are better off (or indifferent) in full unraveling, while those ranked higher are worse off. Inequality (3) indicates that full unraveling tends to benefit medium or low quality firms while hurting high quality firms.

On the workers' side, w_i is better off in full unraveling if

$$i - W + F < \alpha (i - W + F) + (1-\alpha) \left[\frac{(W+1)}{2} - W + F \right], \tag{4}$$

or equivalently,

$$i < \hat{i} \equiv \frac{W+1}{2}. \tag{5}$$

The LHS of (4) is w_i's payoff in an equilibrium without unraveling, since it is matched with firm f_{i-W+F}. In an equilibrium with full unraveling, w_i is matched with firm $j(\hat{r}) = \hat{r} - W + F$, which depends on her type $\hat{r}$ in Period 1. The RHS gives the expected payoff. The function $\hat{i}$ is defined as the cutoff worker, which suggests that full unraveling tends to benefit low quality workers while hurting high quality workers.

Now we consider a banning policy on exploding offers. Suppose every participant holds the common belief that the market does not unravel if exploding offers are banned but fully unravels if they are allowed. Then clearly such a policy will be supported by high quality firms and workers, and opposed by firms of low or medium qualities and workers of low qualities. Similar conclusion is drawn from the experimental results in Niederle and Roth (2009). They find that while early matches are costly for the highest quality firms and workers, some lower quality firms and applicants tend to gain from them. The conclusion also explains

[15] Firms $f_1, f_2, ..., f_{F-W}$ are indifferent since they are unmatched in both cases.

[16] There is an abuse of language here since $\hat{j}$ is not necessarily an integer.

the prevalence of exploding offers in practice and why it is sometimes difficult to achieve consensus on how to solve the unraveling problem in real markets (Niederle and Roth, 2005).

5 Extensions

In a three-period model, I explore whether exploding offers of different time limits may have different effects on market unraveling. Moreover, a different information structure is adopted and a new set of parameters is discussed: the concavity (convexity) of quality distributions over workers and firms.

In the market, there are three firms and three workers with aligned preferences: $f_3 \succ f_2 \succ f_1$ and $w_3 \succ w_2 \succ w_1$. Their qualities are distributed as the following. Firms value a match with worker w_i as v_i: $v_3 = 1$, $v_2 = v$, and $v_1 = \frac{v}{2}$; workers value a match with firm f_j as u_j: $u_3 = 1$, $u_2 = u$, and $u_1 = \frac{u}{2}$, where $u \in [0, 1]$ and $v \in [0, 1]$. Unmatched market participants derive zero utility: $v_0 = u_0 = 0$. Thus, a larger v (u) indicates a more concave quality distribution over workers (firms).

Similar to the basic model, let Θ be the set of all possible strict rankings of workers, in which each ranking/state is realized with equal probability $\frac{1}{6}$. Denote the true state as $\theta \in \Theta$. The market lasts for three periods. The true state θ is revealed in Period 3. At the beginning of Period t, a public signal s_t is observed by both sides, $t = 1, 2$. The public signal now takes form of a worker's "name". If a worker named "i" is ranked r in the true state, denoted as $\theta_r = i$, the probability of the signal being i is given by $\Pr[s_t = i \mid \theta_r = i] = \frac{r}{6}$. The name of a higher-ranked worker in the true state is more likely to be the signal, and the probabilities of all ranks sum up to 1. Two signals s_1 and s_2 are independent.

In addition to the exploding offers that have to be accepted within the same period (denoted as $D = 0$), this three-period model allows us to analyze the ones that can be held for one period (denoted as $D = 1$). Within such a time limit, the information evolves into a different status. The following two propositions discuss the equilibrium without unraveling. Recall by Proposition 1, in a market with only open offers, such an equilibrium is supported by the entire parameter space: $u \in [0, 1]$ and $v \in [0, 1]$. Proposition 5 gives the supporting parameter space when there are no constraints on exploding offers (both $D = 0$ and $D = 1$ are allowed), while Proposition 6 investigates the market where exploding offers can only come with a longer time limit ($D = 1$ is allowed but not $D = 0$).

Proposition 5. *When there are no constraints on exploding offers, the equilibrium without unraveling exists under one of the following conditions: (1) $u \in [0, \frac{10}{33}]$; (2) $u \in (\frac{10}{33}, \frac{4}{11}]$ and $v \in [\frac{10}{23}, 1]$; (3) $u \in (\frac{4}{11}, \frac{18}{31}]$ and $v \in [\frac{6}{11}, 1]$; (4) $u \in (\frac{18}{31}, \frac{18}{23}]$ and $v \in [\frac{18}{23}, 1]$; (5) $u \in (\frac{18}{23}, \frac{4}{5}]$ and $v \in [\frac{18}{19}, 1]$.*

Proposition 5 suggests the equilibrium without unraveling is more likely to sustain when u

is small and v is big, which can be clearly seen in Figure 2. The light shaded area A illustrates the parameter space supporting the equilibrium. As mentioned before, a deviation involves both sides of the market: it requires that a firm wants to make an early offer to a worker who wants to accept it. A smaller u means a more convex quality distribution over firms. Then the top firm is more worth waiting because it is much better than the rest. Thus, an early offer is less likely to be accepted, and a deviation is less likely to succeed. On the other hand, when v is bigger, the quality distribution over workers is more concave. Then the competition for higher-ranked workers becomes less fierce, and an early offer is less likely to be made. A similar intuition is reflected in the following proposition.

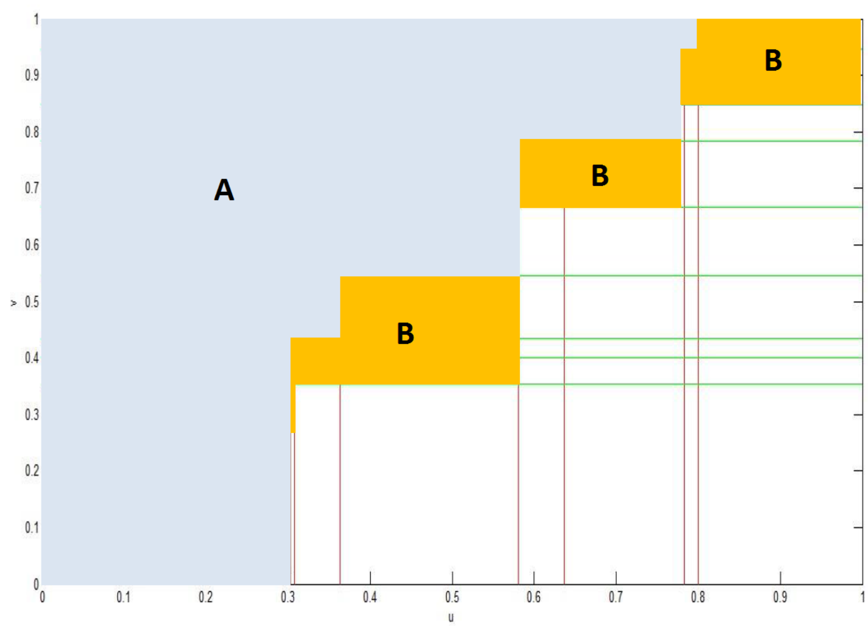


Figure 2: The Equilibrium Without Unraveling

Proposition 6. *When exploding offers that have to be accepted within the same period are not allowed, the equilibrium without unraveling exists under one of the following conditions: (1) $u \in [0, \frac{10}{33}]$; (2) $u \in (\frac{10}{33}, \frac{4}{13}]$ and $v \in [\frac{10}{37}, 1]$; (3) $u \in (\frac{4}{13}, \frac{18}{31}]$ and $v \in [\frac{6}{17}, 1]$; (4) $u \in (\frac{18}{31}, \frac{18}{23}]$ and $v \in [\frac{2}{3}, 1]$; (5) $u \in (\frac{18}{23}, 1]$ and $v \in [\frac{28}{33}, 1]$.*

By comparing the two propositions above, we can see there exists a continuation in the effects of exploding offers. When exploding offers with shorter time limits are allowed, an equilibrium without unraveling is less likely to sustain. Such a result is illustrated in Figure 2. When there are only open offers in market, the entire parameter space supports the equilibrium, which shrinks to the shaded area (A and B) when exploding offers with longer time limits ($D = 1$)

are allowed. Moreover, when the constraints on exploding offers are removed all together, it shrinks further to the light shaded area A.

When exploding offers come with a longer time limit, a better strategy for a worker who has received an early offer is to hold it for one period, instead of responding right away. So no deviation will take place in Period 2 given that such offers will always be held until the last period. For the same reason, when a firm considers making an offer in Period 1, it has to take into account the change of information status during Period 2 and calculate the expected utilities. For example, when $u \in (\frac{18}{31}, \frac{18}{23})$, consider the deviation of f_2 by making an offer to the worker whose name is in s_1. The offer will be rejected if $s_2 = s_1$, i.e., if two signals fall on this same worker; but it will be accepted if $s_2 \neq s_1$. Recall when making an exploding offer with $D = 0$, a firm always knows whether it will be accepted or not. In contrast, when workers are allowed to hold an offer longer, firms are taking on more risks in deviation. This explains the result that unraveling is less likely to happen when the time limits of exploding offers are longer.

6 Conclusion

Many two-sided matching markets tend to unravel in time with transactions occurring earlier and earlier. Using a two-period decentralized model, this paper shows that when a market culture allows firms to make exploding offers, such unraveling is more likely to take place and lead to an unstable matching due to the lack of information in the early stages.

An excess supply of labor makes a market less vulnerable to the presence of exploding offers, while the effect of signal accuracy is ambiguous, depending on how uncertainty is resolved in a market. Therefore, although exploding offers in general tends to facilitate early transactions, it is only a necessary but not a sufficient condition for unraveling to occur. The policy regarding exploding offers should be tailored to the specific environment of interest, while taking into consideration such market characteristics as labor supply and demand and information structure. More importantly, the welfare analysis in this paper indicates that a ban towards exploding offers may benefit high quality firms and workers but hurt agents of low qualities. This suggests the need for policymakers to balance the costs and gains when addressing the unraveling problem in a market.

Admittedly, the model in this paper only provides a simple and tractable benchmark for the analysis of exploding offers. Further extensions into environments like heterogeneous preferences or asymmetric information may give us some additional insights regarding how the use of exploding offers can affect the outcome of a matching market.

Appendix: Proofs

Proof of Proposition 1

Proof. Consider a worker who has received an early open offer from firm f_j $(j = 1, 2, ..., F-1)$. As long as a better firm $f_{j'}$ $(j' > j)$ moves in a later period in equilibrium, she strictly prefers holding to accepting or rejecting the offer right away.

Firstly, holding is preferred to accepting. By choosing to hold but not to accept, the worker is strictly better off if she receives a better offer in a later period. If not, she is not worse off since she still has the open offer from f_j. Secondly, holding is preferred to rejecting. By choosing to hold but not to reject, the worker is not worse off if she receives a better offer in a later period; she is strictly better off otherwise.

Knowing this, the best firm f_F strictly prefers to wait until the last period, so that all workers will stay in the market and the one of the best quality can be perfectly identified. Since no offer is accepted in Period 1, the other firms cannot make themselves better off by moving early; instead, they incur the risk of being rejected in the last period and remaining unmatched.

Hence, there always exist profitable deviations from an equilibrium with partial or full unraveling. Only an equilibrium without unraveling can sustain. It yields the assortative matching according to the true ranking of workers $\succ$, which is the unique stable matching in the current environment with strict rankings and aligned preferences. $\square$

Proof of Lemma 1

Proof. When making an offer in Period 2, every firm is indifferent because an open offer is equivalent to an exploding offer. Now consider an offer in Period 1. The best firm f_F is still indifferent since neither an exploding offer nor an open offer will be rejected by any worker. However, for other firms, making an open offer is never the strictly best response.

First, when making an exploding offer that has to be accepted within the same period, a firm always knows whether it will be accepted. This is because there is no information asymmetry in the current setting, and the information status remains the same within a period.

Next, if an exploding offer will be accepted, making an exploding offer yields the same or a higher payoff than making an open offer. Suppose a firm makes an open offer in Period 1, there are three possible responses: (i) it is accepted right away; (ii) it is held and accepted in Period 2; (iii) it is held and rejected in Period 2. Compared to an exploding offer, the firm yields the same payoff in cases (i) and (ii), but is strictly worse off in case (iii).

Finally, if an exploding offer will be rejected, waiting yields the same or a higher payoff than making an exploding offer or an open offer. A firm never wants to make an exploding offer knowing it will be rejected because then it cannot make an offer to the same worker again. On the other hand, if a firm makes an open offer in Period 1, there are three possible responses: (i) it is rejected right away; (ii) it is held and accepted in Period 2; (iii) it is held and rejected in Period 2. Compared to waiting, the firm is strictly worse off in cases (i) and (iii), and is weakly worse off in case (ii). □

Proof of Proposition 2

Proof. Below I first calculate the updated beliefs of firms and workers after they observe a signal in Period 1.

After observing a signal $\hat{\succ}$, posteriors on the true state are given by

$$\begin{aligned}\Pr(\hat{\succ} \mid \hat{\succ}) &= \frac{\left[\alpha + (1-\alpha)\frac{1}{W!}\right]\frac{1}{W!}}{\left[\alpha + (1-\alpha)\frac{1}{W!}\right]\frac{1}{W!} + (W!-1)\left[(1-\alpha)\frac{1}{W!}\right]\frac{1}{W!}} \\ &= \alpha + \frac{1-\alpha}{W!}.\end{aligned}$$

For any $\succ' \neq \hat{\succ}$,

$$\Pr(\succ' \mid \hat{\succ}) = \frac{1 - \frac{\alpha W! + (1-\alpha)}{W!}}{W!-1} = \frac{1-\alpha}{W!}.$$

Posteriors on types are given by

$$\begin{aligned}\Pr(\hat{r} \mid \hat{r}) &= \Pr(\hat{\succ} \mid \hat{\succ}) + [(W-1)!-1]\Pr(\succ' \mid \hat{\succ}) \\ &= \alpha + \frac{1-\alpha}{W},\end{aligned}$$

and $\forall r' \neq \hat{r}$

$$\Pr(r' \mid \hat{r}) = \frac{1 - \left(\alpha + \frac{1-\alpha}{W}\right)}{W-1} = \frac{1-\alpha}{W}.$$

Thus, the expected quality of a type-$\hat{r}$ worker is

$$EV(\hat{r}) = \left(\alpha + \frac{1-\alpha}{W}\right)\hat{r} + \sum_{r' \neq \hat{r}} \frac{1-\alpha}{W} r' = \alpha\hat{r} + \frac{(1-\alpha)(W+1)}{2}.$$

Next, consider the case where $W > F$. In an equilibrium without unraveling, no actions are taken in Period 1. In Period 2, after $\succ$ is revealed, f_F makes an offer to w_W, f_{F-1} to w_{W-1}, ..., and f_1 to w_{W-F+1}. All offers are accepted.

It is clear that workers do not have any incentive to deviate, nor does firm f_F. Given all others are playing the equilibrium strategy, a firm f_j with $j = 1, 2, ..., F-1$ will not deviate and make an offer to a different worker in Period 2, since it will not be accepted by a worker better than its current match w_{j-F+W}. I now focus on checking the deviation of f_j in Period

1. Such a deviation involves both sides of the market: a firm should want to make an early exploding offer to a worker who wants to accept it. So a sufficient condition for the existence of an equilibrium without unraveling is that, for each worker type $\hat{r}$, the firms whose offer would be accepted are not willing to offer.

Suppose f_j deviates by making an early offer to a type-$\hat{r}$ worker in Period 1, and $\hat{r} \neq \hat{r}(j)$. The offer is accepted if

$$j \geq \left(\alpha + \frac{1-\alpha}{W}\right) \times j(\hat{r}) + \frac{1-\alpha}{W} \times \left(\sum_{r'=W-F+1}^{W} j(r') - j(\hat{r})\right) - \frac{1-\alpha}{W} \times 1, \tag{6}$$

or equivalently,

$$j \geq \underline{j_1^A}(\hat{r}) \equiv \alpha \times (\hat{r} - W + F) + \frac{(1-\alpha)\left(F^2 + F - 2\right)}{2W}. \tag{7}$$

The function $\underline{j_1^A}(\hat{r})$ is defined as the lowest ranked firm that is accepted by type $\hat{r}$. On the other hand, if accepted, the firm is willing to make such an offer if

$$j + W - F \leq EV(\hat{r}), \tag{8}$$

or equivalently,

$$j \leq \overline{j_1^O}(\hat{r}) \equiv \alpha\hat{r} + \frac{(1-\alpha)(W+1)}{2} - W + F. \tag{9}$$

The function $\overline{j_1^O}(\hat{r})$ is defined as the highest ranked firm that wants to make an early offer to type $\hat{r}$. The sufficient condition for no deviation in this case is that for each type, there does not exist a firm that is willing to offer, and is accepted. That is, $\forall \hat{r}$, we need to have

$$\underline{j_1^A}(\hat{r}) \geq \overline{j_1^O}(\hat{r}) \text{ and } \overline{j_1^O}(\hat{r}) \geq 1, \tag{10}$$

or

$$\overline{j_1^O}(\hat{r}) < 1, \tag{11}$$

which solves

$$W \geq 2 + F. \tag{12}$$

(10) ensures that there does not exist a j such that $\underline{j_1^A}(\hat{r}) \leq j \leq \overline{j_1^O}(\hat{r})$. (11) is a boundary condition where no firms are willing to make an offer to a type-$\hat{r}$ worker.[17]

Now we consider the deviation of a firm f_j to its signal-suggested type $\hat{r}(j)$. The offer is

[17] Another boundary case would be that a type-$\hat{r}$ worker does not accept any offer, which never holds because f_F will always be accepted, that is, $\underline{j_1^A}(\hat{r}) < F, \forall \hat{r}$.

accepted if

$$j \geq \left(\alpha + \frac{1-\alpha}{W}\right) \times j(\hat{r}(j)) + \frac{1-\alpha}{W} \times \left(\sum_{r'=W-F+1}^{W} j(r') - j(\hat{r}(j))\right) - \left(\alpha + \frac{1-\alpha}{W}\right), \quad (13)$$

or equivalently,

$$j \geq \underline{j_2^A} \equiv \frac{F^2+F-2}{2W} - \frac{\alpha}{1-\alpha}. \quad (14)$$

If accepted, the firm is willing to make such an offer if

$$j + W - F \leq EV(\hat{r}(j)), \quad (15)$$

or equivalently,

$$j \leq \overline{j_2^O} \equiv \frac{1}{2} + F - \frac{W}{2}. \quad (16)$$

The sufficient condition for no deviation in this case is that there does not exist a firm willing to make an offer to its signal-suggested type, and is accepted. That is,

$$\underline{j_2^A} \geq \overline{j_2^O} \text{ and } \overline{j_2^O} \geq 1,$$

or

$$\overline{j_2^O} < 1.$$

Combining with (12), the two sufficient conditions when $W > F$ are given by (i) $W \geq 2F$, or (ii) $F + 2 < W \leq 2F - 1$ and $\alpha \leq \frac{(W-F)^2-W+F-2}{(W-F)^2+W+F-2}$.

Now I move on to the case where $W \leq F$. In an equilibrium without unraveling, no actions are taken in Period 1. In Period 2, after $\succ$ is revealed, f_F makes an offer to w_W, f_{F-1} to w_{W-1}, ..., and f_{F-W+1} to w_1. All these offers are accepted.

Suppose a firm f_j deviates by making an early offer to its signal-suggested type $\hat{r}(j) = j - F + W$. The offer is accepted if

$$j \geq \left(\alpha + \frac{1-\alpha}{W}\right) \times j(\hat{r}(j)) + \frac{1-\alpha}{W} \times \left(\sum_{r'=1}^{W} j(r') - j(\hat{r}(j))\right) - \left(\alpha + \frac{1-\alpha}{W}\right), \quad (17)$$

or equivalently,

$$j \geq \underline{j_3^A} \equiv \frac{2F-W+1}{2} - \frac{1}{W} - \frac{\alpha}{1-\alpha}. \quad (18)$$

The firm wants to make such an offer if

$$j + W - F \leq EV(\hat{r}(j)), \quad (19)$$

or equivalently,

$$j \leq \overline{j_3^O} \equiv \frac{1}{2} + F - \frac{W}{2}. \tag{20}$$

It is easy to show that $\underline{j_3^A} < \overline{j_3^O}$, that is, the sufficient condition for no deviation never holds for $W \leq F$.

Therefore, the equilibrium without unraveling always exists if (i) $W \geq 2F$, or (ii) $F + 2 < W \leq 2F - 1$ and $\alpha \leq \frac{(W-F)^2 - W + F - 2}{(W-F)^2 + W + F - 2}$. □

Proof of Proposition 3

Proof. For a deviation from the equilibrium without unraveling to occur, there must exist a type of worker and a firm in Period 1 such that, the firm is willing to offer and the worker is willing to accept.

When $W > F$, for a deviation between a non-signal-suggested pair to exist, we need $\exists \hat{r}$ such that

$$\overline{j_1^O}(\hat{r}) - \underline{j_1^A}(\hat{r}) > 1 \text{ if } \underline{j_1^A}(\hat{r}) \geq 0, \tag{21}$$

or

$$\overline{j_1^O}(\hat{r}) > 1 \text{ if } \underline{j_1^A}(\hat{r}) < 0. \tag{22}$$

(21) ensures that the range between $\overline{j_1^O}(\hat{r})$ and $\underline{j_1^A}(\hat{r})$ is larger than 1, so that there always exists an integer in between. (22) is a boundary case where $\underline{j_1^A}(\hat{r}) < 0$. Then we need the range to be even larger so that at least f_1 is willing to offer.[18] It is easy to confirm that the two conditions above never hold.

Similarly, for a deviation between a signal-suggested pair to exist, we need

$$\overline{j_2^O} - \underline{j_2^A} > 1 \text{ if } \underline{j_2^A} \geq 0, \tag{23}$$

or

$$\overline{j_2^O} > 1 \text{ if } \underline{j_2^A} < 0, \tag{24}$$

which solves $F < W < 2F - 1$ and $\alpha > \frac{(W-F)^2 + W + F - 2}{(W-F)^2 + 3W + F - 2}$.

When $W \leq F$, for a deviation between a signal-suggested pair to exist, we need

$$\overline{j_3^O} - \underline{j_3^A} > 1 \text{ if } \underline{j_3^A} \geq 0, \tag{25}$$

or

$$\overline{j_3^O} > 1 \text{ if } \underline{j_3^A} < 0, \tag{26}$$

which solves $W \leq F$ and $\alpha > \frac{W-1}{2W-1}$.

[18] There is another boundary where $\overline{j_1^O}(\hat{r}) > F$, which never holds since f_F never wants to deviate.

Therefore, the equilibrium without unraveling never exists if (i) $W \leq F$ and $\alpha > \frac{W-1}{2W-1}$; or (ii) $F < W < 2F-1$ and $\alpha > \frac{(W-F)^2+W+F-2}{(W-F)^2+3W+F-2}$. □

Proof of Proposition 4

Proof. Consider the case where $W > F$.

In an equilibrium with full unraveling, after $\hat{\succ}$ is revealed in Period 1, f_F makes an offer to type $\hat{r} = W$, f_{F-1} to $\hat{r} = W-1$, ..., and f_1 to $\hat{r} = W-F+1$. All offers are accepted. Such an equilibrium can never sustain because after Period 1, there are $W-F$ workers left in the market. Given all the other firms move early, each firm has an incentive to deviate to Period 2, in which case it becomes the only firm left in the market and can choose the best remaining worker.

Consider the case where $W \leq F$.

In an equilibrium with full unraveling, after $\hat{\succ}$ is revealed in Period 1, f_F makes an offer to type $\hat{r} = W$, f_{F-1} to $\hat{r} = W-1$, ..., and f_{F-W+1} to $\hat{r} = 1$. All these offers are accepted. No workers are left in the market after the first period. Therefore, a firm has no incentive to deviate as long as in its deviation, no worker would reject her current offer and become available in Period 2. That is, in the subgame after any firm's deviation, all workers still accept their offers in Period 1.

Suppose a firm $f_{j'}$ deviates and waits until Period 2, $j' = F-W+1, F-W+2, ..., F$. A worker of type $\hat{r}$ would still accept her current offer if $j' \leq j(\hat{r})$, that is, a worker would never deviate for a firm that is worse than her offer in equilibrium, which is from her signal-suggested firm $j(\hat{r})$. The binding condition for the existence of an equilibrium with full unraveling then requires type $\hat{r} = 1$ not to unilaterally reject her offer in Period 1 in the deviation of f_F, that is,

$$F - W + 1 \geq \frac{1-\alpha}{2}F + \left(\alpha + \frac{1-\alpha}{2}\right)(F-W). \tag{27}$$

The RHS of (27) is the worker's payoff if she accepts her offer in equilibrium. The LHS is the worker's expected payoff if she rejects. In this case, after the first period, there are two workers ($\hat{r} = 1$ and $\hat{r} = W$) and $F-W+2$ firms (f_F, f_{F-W+1}, f_{F-W}, f_{F-W-1}, ..., and f_1) left in the market. Type $\hat{r} = 1$ is matched with f_F if she turns out to have a higher quality than $\hat{r} = W$, and is matched with f_{F-W} otherwise since f_{F-W+1} is no longer available to her after the rejection. Compared to a higher type, the probability of a worker having a higher quality in the true state is given by $\frac{1-\alpha}{W!} \times \frac{W!}{2} = \frac{1-\alpha}{2}$. Equation (27) solves

$$\alpha \geq \frac{W-2}{W}.$$

Together with the constraint $W \leq F$, an equilibrium with full unraveling exists if $W \leq F$ and $\alpha \geq \frac{W-2}{W}$.

On the other hand, when $W \le F$ and $\alpha < \frac{W-2}{W}$, an equilibrium with full unraveling never exists since type $\hat{r} = 1$ has an incentive to reject her offer in Period 1 in the deviation of f_F, which makes f_F strictly prefer to deviate. Combined with the fact that such an equilibrium never exists when $W > F$, we obtain the sufficient and necessary condition for the existence of an equilibrium with full unraveling: $W \le F$ and $\alpha \ge \frac{W-2}{W}$. □

Sketch of Proof of Proposition 5

Proof. Under this framework, there are two types in Period 1. I denote the worker whose name is in s_1 as type $[s_1]$, and all the others as $[-s_1]$. In Period 2, depending on whether s_2 is the same as s_1, four possible types may appear. If two signals fall on the same worker ($s_1 = s_2 = s$), that worker is denoted as type $[s, s]$, and all the others as $[-s, -s]$. If two signals fall on different workers, both of them are denoted as type $[s_1, s_2]$, and all the others as $[-s_1, -s_2]$.

After observing a signal s_1 in Period 1, posteriors on the true state are given by

$$
\begin{aligned}
& \Pr\left[\theta_r = i \mid s_1 = i\right] \\
= \; & \frac{\Pr\left[s_1 = i \mid \theta_r = i\right] \Pr\left[\theta_r = i\right]}{2\Pr\left[s_1 = i \mid \theta_1 = i\right] \Pr\left[\theta_1 = i\right] + 2\Pr\left[s_1 = i \mid \theta_2 = i\right] \Pr\left[\theta_2 = i\right] + 2\Pr\left[s_1 = i \mid \theta_3 = i\right] \Pr\left[\theta_3 = i\right]} \\
= \; & \frac{\frac{r}{6} \times \frac{1}{6}}{2 \times \frac{1}{6} \times \frac{1}{6} + 2 \times \frac{2}{6} \times \frac{1}{6} + 2 \times \frac{3}{6} \times \frac{1}{6}} \\
= \; & \frac{r}{12}.
\end{aligned}
$$

Posteriors on types are then $\Pr\left(r^{\theta}_{[s_1]} = r \mid s_1\right) = 2 \times \frac{r}{12} = \frac{r}{6}$, and $\Pr\left(r^{\theta}_{[-s_1]} = r \mid s_1\right) = \frac{1-\frac{r}{6}}{2} = \frac{6-r}{12}$. In a market where two signals fall on the same worker, after observing s_1 and s_2, posteriors on the true state are

$$
\begin{aligned}
& \Pr\left[\theta_r = i \mid s_1 = s_2 = i\right] \\
= \; & \frac{\frac{r}{6} \times \frac{r}{6} \times \frac{1}{6}}{2 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} + 2 \times \frac{2}{6} \times \frac{2}{6} \times \frac{1}{6} + 2 \times \frac{3}{6} \times \frac{3}{6} \times \frac{1}{6}} \\
= \; & \frac{r^2}{28}.
\end{aligned}
$$

Posteriors on types are

$$
\Pr\left(r^{\theta}_{[s,s]} = r \mid s_1 = s_2 = s\right) = 2 \times \frac{r^2}{28} = \frac{r^2}{14},
$$

and

$$\Pr\left(r^{\theta}_{[-s,-s]} = r \mid s_1 = s_2 = s\right) = \frac{1-\frac{r^2}{14}}{2} = \frac{14-r^2}{28}.$$

In a market where two signals fall on different workers, after observing s_1 and s_2, posteriors on the true state are

$$\begin{aligned} & \Pr\left[\theta_r = i \mid s_1 = i, s_2 = j\right] \\ = & \frac{\frac{r}{6} \times \left(1-\frac{r}{6}\right) \times \frac{1}{6}}{2 \times \frac{1}{6} \times \left(1-\frac{1}{6}\right) \times \frac{1}{6} + 2 \times \frac{2}{6} \times \left(1-\frac{2}{6}\right) \times \frac{1}{6} + 2 \times \frac{3}{6} \times \left(1-\frac{3}{6}\right) \times \frac{1}{6}} \\ = & \frac{r \times (6-r)}{44}. \end{aligned}$$

Posteriors on types are

$$\Pr\left(r^{\theta}_{[s_1 \mathrm{or} s_2]} = r \mid s_1 \neq s_2\right) = 2 \times \frac{r \times (6-r)}{44} = \frac{r\,(6-r)}{22},$$

and

$$\Pr\left(r^{\theta}_{[-s_1,-s_2]} = r \mid s_1 \neq s_2\right) = 1 - 2 \times \frac{r\,(6-r)}{22} = \frac{22 - 2r\,(6-r)}{22}.$$

Thus, the expected qualities are $E_{[s_1]} = \frac{6+5v}{12}$, $E_{[-s_1]} = \frac{6+13v}{24}$, $E_{[s,s]} = \frac{18+9v}{28}$, $E_{[-s,-s]} = \frac{10+33v}{56}$, $E_{[s_1,s_2]} = \frac{18+21v}{44}$, and $E_{[-s_1,-s_2]} = \frac{2+6v}{11}$.

The main idea of the rest of the proof is to use backward induction, which means, for each possible deviation, the subgames afterwards are solved and the expected utility is calculated. For the equilibrium to sustain, we simply need to ensure the expected utility is higher on the equilibrium path. Because the number of subgames is very big for this game, I provide a sketch of proof to avoid tediousness on presentation.

In an equilibrium without unraveling, the best firm f_3 has no incentive to deviate.

Step 1: Consider the possible deviations in Period 1. No deviation requires $v \geq \frac{6}{11}$ or $u \leq \frac{4}{11}$.

f_1: If accepted, f_1 always wants to deviate since it is getting the worst worker in equilibrium. However, both types $[s_1]$ and $[-s_1]$ will reject.

f_2: If accepted, f_2 is willing to make an offer to $[s_1]$ if $v < E_{[s_1]} = \frac{6+5v}{12}$, i.e., $v < \frac{6}{7}$. However, $[s_1]$ will not accept.[19] If accepted, f_2 is willing to make an offer to $[-s_1]$ if $v < E_{[-s_1]} = \frac{6+13v}{24}$, i.e., $v < \frac{6}{11}$. In this case, type $[-s_1]$ will not reject such an offer if $u > \frac{4}{11}$.

Step 2: Consider the possible deviation in Period 2 when two signals fall on the same worker ($s_1 = s_2 = s$). No deviation requires ($v \geq \frac{18}{19}$ or $u \leq \frac{18}{23}$), and ($v \geq \frac{10}{23}$ or $u \leq \frac{10}{33}$).

f_1: If accepted, f_1 always wants to deviate. But both $[s,s]$ and $[-s,-s]$ will reject.

f_2: If accepted, f_2 is willing to make an offer to $[s,s]$ if $v < E_{[s,s]} = \frac{18+9v}{28}$, i.e., $v < \frac{18}{19}$. $[s,s]$

[19] When there exists multiple equilibria, the condition holds as long as the worker rejects in one of them.

accepts if $u > \frac{18}{23}$. If accepted, f_2 is willing to make an offer to $[-s,-s]$ if $v < E_{[-s,-s]} = \frac{10+33v}{56}$, i.e., $v < \frac{10}{23}$. Type $[-s,-s]$ will not reject such an offer if $u > \frac{10}{33}$.

Step 3: Consider the possible deviation in Period 2 when two signals fall on different workers ($s_1 \neq s_2$). No deviation requires $u \leq \frac{4}{5}$, ($v \geq \frac{18}{23}$ or $u \leq \frac{18}{31}$), and ($v \geq \frac{2}{5}$ or $u \leq \frac{4}{13}$).

f_1: If accepted, f_1 always wants to deviate. $[s_1, s_2]$ will always reject. $[-s_1, -s_2]$ accepts if $u > \frac{4}{5}$.

f_2: If accepted, f_2 is willing to make an offer to $[s_1, s_2]$ if $v < E_{[s_1,s_2]} = \frac{18+21v}{44}$, i.e., $v < \frac{18}{23}$. $[s_1, s_2]$ accepts if $u > \frac{18}{31}$. If accepted, f_2 is willing to make an offer to $[-s_1, -s_2]$ if $v < E_{[-s_1,-s_2]} = \frac{2+6v}{11}$, i.e., $v < \frac{2}{5}$. Type $[-s_1, -s_2]$ will not reject such an offer if $u > \frac{4}{13}$.

Step 4:

Combining Step 1 and 2, the condition for no deviations in a market with $s_1 = s_2$ is given by $u \in [0, \frac{10}{33}]$ or ($u \in (\frac{10}{33}, \frac{4}{11}]$ and $v \in [\frac{10}{23}, 1]$) or ($u \in (\frac{4}{11}, \frac{18}{31}]$ and $v \in [\frac{6}{11}, 1]$) or ($u \in (\frac{18}{31}, \frac{18}{23}]$ and $v \in [\frac{10}{23}, 1]$) or $v \in [\frac{18}{19}, 1]$.

Combining Step 1 and 3, the condition for no deviations in a market with $s_1 \neq s_2$ is given by $u \in [0, \frac{4}{13}]$ or ($u \in (\frac{4}{13}, \frac{4}{11}]$ and $v \in [\frac{2}{5}, 1]$) or ($u \in (\frac{4}{11}, \frac{18}{31}]$ and $v \in [\frac{6}{11}, 1]$) or ($u \in (\frac{18}{31}, \frac{4}{5}]$ and $v \in [\frac{18}{23}, 1]$).

To sum up, for an equilibrium without unraveling to exist in either of the cases above, we need either one of the following conditions to hold: (1) $u \in [0, \frac{10}{33}]$; (2) $u \in (\frac{10}{33}, \frac{4}{11}]$ and $v \in [\frac{10}{23}, 1]$; (3) $u \in (\frac{4}{11}, \frac{18}{31}]$ and $v \in [\frac{6}{11}, 1]$; (4) $u \in (\frac{18}{31}, \frac{18}{23}]$ and $v \in [\frac{18}{23}, 1]$; (5) $u \in (\frac{18}{23}, \frac{4}{5}]$ and $v \in [\frac{18}{19}, 1]$. $\square$

Sketch of Proof of Proposition 6

Proof. The method used here is similar to the proof of Proposition 5, except that the workers now can hold an exploding offer for one period. Then a dominate strategy for a worker who has received an early offer is to hold it for one period, instead of responding right away. Hence, for the same reason as that in Proposition 1, no deviation will take place in Period 2 given that all offers in Period 2 will be held until the last period.

On the other hand, in the subgame following a deviation in Period 1, we need to consider how worker types evolve in order to calculate expected utilities. A type-$[s_1]$ worker in Period 1 will turn into type $[s, s]$ with probability $\frac{7}{18}$, and into type $[s_1, s_2]$ with probability $\frac{11}{18}$ in Period 2. A type-$[-s_1]$ worker in Period 1 will turn into type $[s_1, s_2]$ with probability $\frac{11}{36}$, into type $[-s,-s]$ with probability $\frac{14}{36}$, and into type $[-s_1, -s_2]$ with probability $\frac{11}{36}$.

In an equilibrium without unraveling, the best firm f_3 has no incentive to deviate.

Step 1: No deviation by f_1 requires $u \leq \frac{4}{5}$ or $v \geq \frac{8}{15}$. If accepted, f_1 is always willing to deviate and make an offer in Period 1. After holding the early offer from f_1 till Period 2, type $[-s_1, -s_2]$ will accept if $u > \frac{4}{5}$; the offer will be rejected in all other cases. Therefore, f_1 never offers to $[s_1]$ in Period 1, and never offers to $[-s_1]$ if $u < \frac{4}{5}$. When $u > \frac{4}{5}$, f_1 will not offer to

$[-s_1]$ if $\frac{v}{2} > \frac{14}{36} \times \frac{8v}{28} + \frac{11}{36} \times \frac{2+6v}{11} + \frac{11}{36} \times \frac{17v}{44}$, where $\frac{17v}{44}$, $\frac{8v}{28}$, and $\frac{2+6v}{11}$ are the firm's expected utilities when $[-s_1]$ becomes $[s_1, s_2]$, $[-s, -s]$, and $[-s_1, -s_2]$ respectively. It solves $v > \frac{8}{15}$.

Step 2: Consider the deviation of f_2 by making an offer to type $[s_1]$. No deviation requires $u \leq \frac{18}{31}$, ($u \in (\frac{18}{31}, \frac{18}{23}]$ and $v \geq \frac{2}{3}$), or ($u > \frac{18}{23}$ and $v \geq \frac{28}{33}$).

If $u < \frac{18}{31}$, the offer will always be rejected in Period 2.

If $u \in (\frac{18}{31}, \frac{18}{23})$, the offer will be accepted when $[s_1]$ becomes $[s_1, s_2]$, and will be rejected when $[s_1]$ becomes$[s, s]$. Therefore, f_2 will not offer if $v > \frac{7}{18} \times \frac{24v}{28} + \frac{11}{18} \times \frac{18+21v}{44}$, i.e., $v > \frac{2}{3}$.

If $u > \frac{18}{23}$, the offer will always be accepted in Period 2. Hence, f_2 will not offer if $v > \frac{28}{33}$.

Step 3: Consider the deviation of f_2 by making an offer to type $[-s_1]$. No deviation requires $u \leq \frac{18}{31}$, ($u \in (\frac{18}{31}, \frac{18}{23}]$ and $v \geq \frac{2}{3}$), or ($u > \frac{18}{23}$ and $v \geq \frac{28}{33}$). requires $u \leq \frac{10}{33}$, ($u \in (\frac{10}{33}, \frac{4}{13}]$ and $v \geq \frac{10}{37}$), ($u \in (\frac{4}{13}, \frac{18}{31}]$ and $v \geq \frac{6}{17}$), or ($u > \frac{18}{31}$ and $v \geq \frac{6}{11}$).

If $u < \frac{10}{33}$, the offer will always be rejected in Period 2.

If $u \in (\frac{10}{33}, \frac{4}{13})$, the offer will be rejected when $[-s_1]$ becomes $[s_1, s_2]$, will be accepted when $[-s_1]$ becomes $[-s, -s]$, and will be rejected when $[-s_1]$ becomes $[-s_1, -s_2]$. Hence, f_2 will not offer to $[-s_1]$ if $v > \frac{11}{36} \times \frac{18v}{22} + \frac{14}{36} \times \frac{10+33v}{56} + \frac{11}{36} \times \frac{19v}{22}$, i.e., $v > \frac{10}{37}$.

If $u \in (\frac{4}{13}, \frac{18}{31})$, the offer will be rejected when $[-s_1]$ becomes $[s_1, s_2]$, will be accepted when $[-s_1]$ becomes $[-s, -s]$, and will be accepted when $[-s_1]$ becomes $[-s_1, -s_2]$. Therefore, f_2 will not offer to $[-s_1]$ if $v > \frac{11}{36} \times \frac{18v}{22} + \frac{14}{36} \times \frac{10+33v}{56} + \frac{11}{36} \times \frac{2+6v}{11}$, i.e., $v > \frac{6}{17}$.

If $u > \frac{18}{31}$, the offer will always be accepted in Period 2. Hence, f_2 will not offer to s_1 if $v > \frac{11}{36} \times \frac{18+21v}{44} + \frac{14}{36} \times \frac{10+33v}{56} + \frac{11}{36} \times \frac{2+6v}{11}$, i.e., $v > \frac{6}{11}$.

Step 4: Combining all conditions above, for an equilibrium without unraveling to exist, we need either one of the following conditions to hold: (1) $u \in [0, \frac{10}{33}]$; (2) $u \in (\frac{10}{33}, \frac{4}{13}]$ and $v \in [\frac{10}{37}, 1]$; (3) $u \in (\frac{4}{13}, \frac{18}{31}]$ and $v \in [\frac{6}{17}, 1]$; (4) $u \in (\frac{18}{31}, \frac{18}{23}]$ and $v \in [\frac{2}{3}, 1]$; (5) $u \in (\frac{18}{23}, 1]$ and $v \in [\frac{28}{33}, 1]$. □

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