Optimal Top-n Policy*

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Abstract

The efficacy of the widely-adopted "top-n" policy in university integration has been questioned because students strategically relocate to low-achieving high schools. We show that when different SES groups have heterogenous relocation costs, the policy can even segregate minorities from the target university, compared to the school-blind policy. A suitably chosen eligibility requirement, featuring the minimum time students must spend at a high school in order to be eligible for top-n admissions, can restore the efficacy of this policy. However, the most stringent requirement is not always optimal. The optimal requirement depends on the original distribution of students across high schools.

Keywords: top-n policy, college admissions, integration, segregation

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1 Introduction

Over the past few decades, affirmative action has been widely implemented in employment and education contexts while often provoking legal and political controversy. Such policies are designed to increase the representation of disadvantaged groups in public spheres and

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to close the socioeconomic gaps that exist between different population groups. Efforts to close these gaps are typically justified based on the fact that they exist as a result of historic discrimination. For example, in K-12 public education, various kinds of quota-based school choice systems (namely "controlled school choice") are used in many big cities in the US. In higher education, many university admissions committees explicitly state their preferences for a student body with a diversity of gender, ethnicity, and socioeconomic status (SES). In Australia, universities are increasingly using so-called "alternative pathways," giving students access to higher education via avenues other than standardized exam scores.

One of the prevailing affirmative action policies in the university sector is the so-called "top-n" policy: a university guarantees admission to students who are in the top n fraction (i.e., $100n$ percent) of graduates in their high school, usually judged by some measure of their educational achievement. The aim is to increase the representation of low-SES students at prestigious universities, since disadvantaged high schools would not typically send many students to such universities. In the US, Texas, California, and Florida have mandated their state universities, including flagship institutions, to guarantee admission to the top n fraction of students in each high school, with the state government specifying the fraction n. Several Group of Eight universities in Australia, such as the Australian National University and Monash University, use this policy as part of their alternative admission pathways.

Contrary to expectations, however, the empirical literature has questioned the efficacy of the top-n policy in creating integrated student bodies at target universities, both in the short run (Kain et al., 2005; Long and Tienda, 2010) and in the long run (Long and Bateman, 2020; Cortes and Klasik, 2020). To explain this puzzle, the literature has further examined student behavior in relation to high school choice, and found that the top-n policy induces strategic enrollment in a high school, by students and their families, or strategic relocation to a new high school (Cullen et al., 2013; Cortes and Friedson, 2014; Estevan et al., 2018). Since the top-n policy gives preferential treatment based on the high school that students attend, rather than their race or gender, marginal students at high-achieving high schools are incentivized to move to lower-achieving schools in order to gain admission to a prestigious university. Intuitively, if such students move high school in order to gain university admission, they will secure the top-n admission slots from low-achieving schools, while displacing some students at these schools who could otherwise have been admitted. As a result, those who initially attend a high-achieving school and then relocate will constitute a large share of the admitted population at the target universities, neutralizing the top-n policy's intended effect.

To examine the equilibrium consequence of this strategic relocation, this paper theoretically studies the effectiveness of the top-n policy by formally modeling the game of student relocation. One of the main features of our model is the heterogeneity of relocation costs among different SES groups. More specifically, we assume that this cost (relative to the utility gain from admission by the target university) is generally higher for low-SES families than for high-SES families, reflecting the former group's financial constraints. Our first result shows that the top-n policy can actually *segregate* low-SES students from the target university when the relocation cost for high-SES families is sufficiently low (Proposition 1). This result is surprising, but also intuitive: since high-SES students are willing to move to any high school while low-SES students may not be so willing, the lowest cutoff across all high schools becomes lower than when the top-n policy is not used, thus enabling high-SES students to occupy a larger share of the university slots.

To mitigate the relocation incentive, policymakers often impose an *eligibility requirement* featuring the minimum time spent at a new high school. That is, students need to attend their new high school for a minimum period of time before graduation in order to be eligible for top-n admission from that school. For example, under the Texas Top Ten (percent) policy, many high schools in Texas stipulate a period of one to two years, so that strategic relocators do not benefit from switching schools just before graduation (Cullen et al., 2013). Since students typically do not prefer to spend an extended period in a low-achieving high school, a longer time requirement implies a higher cost to them.

We then show that, with an appropriate level of eligibility requirement, the top-n policy can achieve a higher degree of integration than the school-blind policy (Proposition 2). This result holds under our assumption that the most stringent eligibility requirement, capable of suppressing any relocation incentives, is feasible. This guarantees that the top-n policy's efficacy is restored when it is deployed together with an effective eligibility requirement.

However, we further show that the degree of integration achieved in equilibrium is not necessarily monotone in the level of eligibility requirement: the most stringent eligibility requirement is *not* always optimal for university integration. To see the intuition, note that the fraction n of the top-n policy is typically not high enough to fill the entire capacity of the target university. The remaining seats are filled through at-large admissions; that is, admissions based on merit, regardless of a student's school identity. In this scenario, the admitted population under the top-n policy is biased toward high-SES groups even when no student moves high school. Perhaps surprisingly, we find that this bias may be corrected at an equilibrium under an intermediate level of eligibility requirement. When the eligibility requirement is not so stringent, students are incentivized to relocate only to a school with characteristics similar enough to their original school. Due to the local relocation, the top-n admissions cutoff of a middle-achieving school can be higher than that of a higher-achieving school in equilibrium. Then, the middle-achieving school may send more students to the target university through at-large admissions under an intermediate requirement than under the most stringent requirement, thereby increasing the share of low-SES students among those admitted. Whether or not this type of equilibrium exists depends on the original distribution of students, and we derive conditions for its existence in terms of equilibrium cutoffs (Propositions 4–5), as well as of model parameters under uniform distributions (Proposition 6). We further offer a numerical example to illustrate when the optimal top-n policy switches between the most stringent and an intermediate levels of eligibility requirement in the parameter space.

1.1 Related literature

The top-n policy has attracted considerable academic attention due to its wide use in real-life contexts. The strand of literature most relevant to our paper discusses how students can strategically respond to the top-n policy when making high school enrollment and relocation decisions.[1]

Cullen et al. (2013) analyze students' high school relocation between the 8th and 10th grades, before and after the introduction of the Texas Top Ten (TTT) policy. They conclude that the change in admissions policies influenced students' high school choices, which consequently increased the proportion of white students in the top 10% pool. Cortes and Friedson (2014) find that the TTT led to property values rising in poorly performing school districts, with the effect strongest in districts with the lowest-performing high schools. This suggests that students strategically relocated to those districts in order to attend a high school with lower-achieving peers.

Estevan et al. (2018) also find direct evidence that students strategically moved schools following the introduction of the TTT. They investigate students between the 9th and 12th grades, and estimate a larger effect than Cullen et al. (2013). The main focus of their paper, however, is the TTT policy's influence on high school integration. Using both theoretical and empirical analyses, the paper shows that although the policy was designed to promote diversity at the college level, the strategic reactions it induced can actually foster greater integration of high schools. Estevan et al. (2018) offer, to the best of our knowledge, the first

[1] As a variant of the top-n policy, the division of university quotas across provinces has also induced a trend of strategic relocation in China, namely "Gaokao migration." See, for example, a news article that discusses this phenomenon, at https://www.globaltimes.cn/content/1149569.shtml (accessed 6 December, 2021.)

theoretical model of strategic relocation under the top-n policy. The main structure of our model follows that of Estevan et al. (2018), but we focus instead on university integration and the role of an eligibility requirement. We also allow for heterogeneous moving costs, which generates potentially adverse effects of the top-n policy, as shown in Proposition 1.

Using a theoretical model in which applicants are selected based on traits that can be endogenously chosen at a cost, Fryer Jr et al. (2003) make the general argument that strategic responses to color-blind affirmative action policies such as top-n may render these policies inefficient over the long run.[2] Our model delivers the disquieting message that the top-n policy can even harm university integration, when students from different SES backgrounds adopt different moving patterns. We also complement the above literature by adding eligibility requirements as a potential solution to the problem of strategic relocation.

These strategic responses may help to reconcile two apparently conflicting findings in the literature: although it has been shown that the top-n policy has widened the pool of high schools that send students to flagship universities (see, for example, Montejano, 2001; Long et al., 2010; and Niu and Tienda, 2010), many studies have questioned its purported achievement of improving diversity at universities.

Investigating the short run, Kain et al. (2005) show that the elimination of race-based affirmative action had a devastating effect on minority enrollment in Texas selective public universities, and that the introduction of the TTT was not effective in countering the effect. Similar conclusions are reached by Long and Tienda (2010). Fletcher and Mayer (2014) find that although the TTT impacted students' college application behavior, there is little evidence that it lead to notable increases in diversity at flagship universities. Combining data from seven flagship universities in Texas, California, and Florida, Long (2007) estimates that the introduction of the top-n policy in these states has offset only one-third of the total losses in minority representation caused by the abolition of race-based affirmative action, and that this effect is not statistically significant.[3]

Investigating the long run, Long and Bateman (2020) analyze the enrollments of 19 flagship universities in nine states following the race-based affirmative action bans. They find that the share of minority students admitted to these universities has been steadily declin-

[2]On a related note, Ellison and Pathak (2021) investigate the consequences of shifting from a race-based affirmative action policy to a color-blind, neighborhood-based alternative at Chicago Public Schools. They conclude that the color-blind option is less effective than racial quotas in increasing minority or low-income access.

[3]Although Niu and Tienda (2010) find evidence that the TTT boosted flagship enrollments among Hispanics in the short run, Harris and Tienda (2012) argue that, after accounting for the increasing percentage of Hispanic students in Texas, Hispanics are more disadvantaged under the top 10% admission regime.

ing, implying that alternative policies, including top-n, have been unable to compensate for the racial diversity loss as a result of the bans. Cortes and Klasik (2020) demonstrate that during the 18 years following enactment of the TTT, it did not result in any meaningful changes to the patterns of admissions from high schools in terms of their racial and ethnic composition, and they are able to provide little to no evidence of its equity-producing effects. Similarly, Kapor (2020) finds that although the TTT has increased enrollments of students from high-poverty schools to flagship universities, it is limited in its ability to increase minority enrollments.

Other effects of the top-n policy have also been discussed in the literature. For example, Andrews et al. (2010), Niu et al. (2006), and Daugherty et al. (2014) investigate the policy's influence on students' college application behavior. Cortes (2010), Bleemer (2021), and Black et al. (2020) focus on its effects on students' graduation and earning outcomes.

Our paper is also related to the extensive literature on race-based affirmative action. Here we focus our review on affirmative action in school choice. In their seminal paper, Abdulkadiroğlu and Sönmez (2003) extend their model to allow for simple affirmative action policies with type-specific quotas. These "hard quotas" (minimum and maximum number of seats allocated to specific type of students) are further analyzed by Kojima (2012), while "soft quotas" (minimum and maximum number of seats are reserved for, in the sense of giving a higher priority to, specific types of students) are introduced and analyzed in Hafalir et al. (2013) and Ehlers et al. (2014).[4] As is common with the school choice problems, the papers in this strand of the literature do not consider the strategic reactions of students to change their type endogenously.

The rest of the paper is organized as follows. Section 2 introduces the model of relocation under the top-n policy. In Section 3, we provide our main results on the optimal top-n policy. Section 4 provides two further discussions on our results. We conclude the paper in Section 5. All proofs are in the Appendix.

2 Model

There is a unit mass of students, each of whom is characterized by an educational achievement $a \in [0, \bar{a}]$ where $\bar{a} > 0$, a socioeconomic status (SES) group g, and a high school s_i in which they are originally enrolled. We consider two SES types, $\bar{g}$ and $\underline{g}$, and three high schools,

[4] Recent studies on controlled school choice and other two-sided matching problems with diversity concerns include Westkamp (2013), Kominers and Sönmez (2016), Fragiadakis and Troyan (2017), Dur et al. (2018), Tomoeda (2018), Aygün and Turhan (2020), and Dur et al. (2020).

s_1, s_2, and s_3.[5] Each high school s_i has a measure q_i of students, where $\sum_{i=1}^{3} q_i = 1$. High schools differ in the distribution of their original students' achievements and SES. They can be ranked in the following way. $F_i(\cdot)$ denotes the distribution of achievements at each school s_i, and we assume that $F_{i+1}(\cdot)$ strictly first-order stochastically dominates $F_i(\cdot)$ for each $i = 1, 2$. p_i^g denotes the share of SES group g in high school s_i, and we assume $p_3^{\bar{g}} \geq p_2^{\bar{g}} \geq p_1^{\bar{g}}$ and $p_3^{\bar{g}} \neq p_1^{\bar{g}}$. These assumptions imply that achievement and SES are correlated in such a way that a high school with a larger index tends to have higher educational achievement and a higher proportion of $\bar{g}$. For the sake of tractability, however, we assume that achievement a and SES g are distributed independently within each high school. Let $p^g := q_1 p_1^g + q_2 p_2^g + q_3 p_3^g$ denote the share of SES group g in the aggregate population.

We consider two universities, U and u. U has a fixed capacity $q_U < 1$ and is preferred by all students. u accommodates all other students; i.e., the capacity of u is $1 - q_U$ or greater. Although we call u a university, u represents any option other than attending the target university U, including attending a less prestigious university or entering the labor market.

Since there is only one competitive university, we focus on U's admissions policy. We consider the *school-blind (SB) policy* and the *top-n policy*. The SB policy admits the top q_U students across all high schools based on their achievement a regardless of their high school identity. By contrast, the top-n policy guarantees admission to any student who is within the top n fraction of their high school peers.

As shown in the literature (Cullen et al., 2013; Estevan et al., 2018), the top-n policy induces strategic relocation across high schools, typically from high-achieving to low-achieving schools. Policymakers are aware of this problem, and in order to mitigate relocation incentives, they often require students to attend a high school long enough to be eligible for top-n admissions from that school. This imposes a higher cost on strategic relocation because students who relocate strategically prefer not to spend an extended period at their new school, which typically has a lower educational achievement than their original school.

To model this type of eligibility requirement, we construct a timeline from 0 to T, where T can be interpreted as the total years of high school. $e \in [0, T]$ denotes the eligibility requirement, which is the minimum time a student is required to spend at a high school in order to be eligible for top-n admissions from that school. When $e = 0$, it means that the top-n policy has no eligibility requirement. We assume that students move at most once in $[0, T]$ and only do so for a strategic reason.[6] Thus, a student is eligible at the new school

[5] As we discuss below, the implications of our main results continue to hold in a more general model. See Section 4.1 for details.

[6] In other words, we implicitly assume that a student's original high school is optimally chosen and thus

only if they relocate before $T-e$, and is otherwise eligible only at their original school. The *top-n policy with an eligibility requirement* e admits students in the following way:

1. (top-n admissions) U admits the top n achievers from each high school. Students are eligible for the top-n admissions at a high school s if they are enrolled in s before $T-e$.

2. (at-large admissions) Among the remaining students, U admits top $q_U - n$ achievers according to their achievement a regardless of their high school identity.

Interestingly, in practice, the level of eligibility requirement e varies across contexts and markets. In the context of the Texas Top Ten policy, Cullen et al. (2013) document that high schools typically impose a requirement of one to two years' attendance in order to "mitigate[s] the scope for gain from late-term transfers during junior or senior year." In Chinese college admissions, the quota of each university is divided among provinces instead of high schools, which has led to a common phenomenon, namely "Gaokao migration," where students strategically migrate across provinces to take the college entrance exam.[7] To curb such incentives, various eligibility requirements are imposed on migrant students wishing to take the college entrance exam in a new province. For example, Sichuan province requires 3 years' residence, and Guizhou requires 12 years, while Henan has no such requirement. Our main research question is to find the optimal level of e by defining the objective function of the policymaker and the relocation game played by students, as we elaborate below.

To model the game of relocation under the top-n policy, we consider the following utility function of students. The utility of a student in SES group $g \in \{\bar{g}, \underline{g}\}$ who relocated from s_i to s_j at time $t \in [0,T]$ and is admitted to U (resp. u) is $v(U) - c_g(|i-j|,t)$ (resp. $v(u) - c_g(|i-j|,t)$). The function $v(\cdot)$ indicates the value of attending a university, and $c_g(|i-j|,t)$ represents the cost of relocation, where $c_g(0,t) = 0$ for any $t \in [0,T]$ without loss of generality. We assume that $(c_{\bar{g}}(\cdot,\cdot), c_{\underline{g}}(\cdot,\cdot))$ satisfies the following five conditions:

(i) $c_g(|i-j|,t)$ is strictly decreasing in t for any $g \in \{\bar{g}, \underline{g}\}$ and $|i-j| > 0$;

(ii) $c_g(|i-j|,t)$ is strictly increasing in $|i-j|$ for any $g \in \{\bar{g}, \underline{g}\}$ and $t \in [0,T]$;

(iii) $c_{\bar{g}}(|i-j|,t) < c_{\underline{g}}(|i-j|,t)$ for any $|i-j| > 0$ and $t \in [0,T]$;

(iv) $c_{\bar{g}}(|i-j|,T) < v(U) - v(u)$ for any $|i-j| > 0$; and

(v) $c_{\bar{g}}(|i-j|,0) > v(U) - v(u)$ for any $|i-j| > 0$.

it is costly to spend an extended period at a different high school.

[7]China's policy is not a percent plan but it can be considered a variant of the top-n policy.

(i) assumes that moving earlier is more costly than moving later. This is a natural assumption given that we focus only on strategic relocation. Since any new high school is less preferable than a student's original school and earlier relocation requires a longer stay at a new high school, it is thus more costly. (ii) means that it is more costly to move to a school that differs more from the original school, where the difference is measured by the school indices. Note that, in our setting, the school index is ordered by the achievement ranking as well as the SES composition. (iii) means that low-SES students tend to have higher costs of relocation. This can also be interpreted as the budget or borrowing constraints, rather than the actual difference in the moving cost. (iv) assumes that the moving cost of a high-SES student is low enough such that they are willing to relocate to any other school just before graduation if this means they can be admitted to U instead of u as a result. This is not necessarily the case for low-SES students; that is, $c_{\underline{g}}(2, T)$ can be higher than $v(U) - v(u)$. Finally, (v) means that no student has an incentive to move at time 0. We adopt this assumption simply in order to render feasible the highest-level eligibility requirement, which enables us to study what happens when relocation incentives are completely suppressed.

We model the strategic relocation induced by the top-n policy as a simultaneous game in which students choose moving strategies $\sigma(a, g, s_i) \in \{s_1, s_2, s_3\} \times [0, T]$. $\sigma(a, g, s_i)$ specifies a pure action of when to move, and which school to move to, for each student with characteristics (a, g, s_i). An equilibrium is σ such that for all $(g, s_i) \in \{\bar{g}, \underline{g}\} \times \{s_1, s_2, s_3\}$ and almost all a in the support of $F_i(\cdot)$, $\sigma(a, g, s_i)$ is a best response to σ. An equilibrium outcome is characterized by a cutoff score at each high school, representing the lowest possible achievement that a student needs in order to be admitted to U from that school. We use π to denote the policy being considered, which can be the SB policy or the top-n policy with requirement $e \in [0, T]$; that is, $\pi \in \{SB\} \cup [0, T]$. $a_i(\pi, \sigma)$ denotes the cutoff score of high school s_i when σ is played under policy π. We show in Appendix A that the existence of an equilibrium is guaranteed for the top-n policy with any eligibility requirement $e \in [0, T]$.

The objective of the policymaker is to achieve an SES composition of the students in U (and u) that is representative; i.e., as close as possible to the composition in the aggregate population. To evaluate this goal, we use a segregation measure of universities. Recall that p_i^g is the share of SES group g in high school s_i and p^g is SES group g's share in the aggregate population. Let $p = (p^{\bar{g}}, p^{\underline{g}})$ be the composition vector in the population and use $p_U(\pi, \sigma)$ (resp. $p_u(\pi, \sigma)$) to denote the SES composition vector of university U (resp. u) when σ is played under policy $\pi \in \{SB\} \cup [0, T]$. The *degree of university segregation under* (π, σ) is defined by

$$\mathcal{I}(\pi, \sigma) := A_1(p) - A_2(p)\left[q_U H(p_U(\pi, \sigma)) + (1 - q_U) H(p_u(\pi, \sigma))\right],$$

where $A_2(p) > 0$ and $H(\cdot)$ is strictly concave. As a special case, if $H(\cdot)$ and $A_1(\cdot)$ are entropy functions and $A_2(p) = 1$, then $\mathcal{I}$ is the *mutual information index (MII)*:

$$\mathcal{I}(\pi, \sigma) = -\sum_g p^g \log(p^g) - \left[-q_U \sum_g p_U^g(\pi, \sigma) \log(p_U^g(\pi, \sigma)) - (1 - q_U) \sum_g p_u^g(\pi, \sigma) \log(p_u^g(\pi, \sigma)) \right].$$

While we use MII for numerical examples, our theoretical analyses apply to any $A_1(\cdot)$, any $A_2(\cdot) > 0$, and any strictly concave function $H(\cdot)$.[8] We omit σ from (π, σ) when the equilibrium is unique under policy π.

Based on the segregation index $\mathcal{I}$, we define the relationship between policies in terms of university integration.

Definition 1. A policy $\pi \in \{SB\} \cup [0, T]$ *achieves a weakly higher (resp. lower) degree of integration* than another policy $\pi' \in \{SB\} \cup [0, T]$ if $\mathcal{I}(\pi, \sigma) \leq \mathcal{I}(\pi', \sigma')$ (resp. $\mathcal{I}(\pi, \sigma) \geq \mathcal{I}(\pi', \sigma')$) for any equilibrium σ under π and any equilibrium σ' under π'.

For the purpose of this paper, we focus only on integration as a policy goal and characterize a policy that achieves the highest level of integration.

Definition 2. A top-n policy $\pi \in [0, T]$ is *an optimal top-n policy* if π achieves a weakly higher degree of integration than any other top-n policy $\pi' \in [0, T]$.

3 Results

3.1 Relocation incentives and market-clearing conditions

In this section, we use a simple example to illustrate the equilibrium analysis; that is, we discuss which students have the incentives to move to which high schools, and how market-clearing conditions determine equilibrium cutoff scores. This example demonstrates that while the top-n policy can achieve integration without student relocation, it can actually exacerbate the segregation problem when students can strategically move across high schools and have heterogeneous moving costs.

Example 1. Suppose that originally each high school s_i has the same measure of students $q_i = \frac{1}{3}$. At each high school, student achievement is uniformly distributed with density

[8] This formulation is consistent with many other segregation indices including Theil's information index (Theil and Finizza, 1971), the variance ratio index (James and Taeuber, 1985), and the Bell-Robinson index (Kremer and Maskin, 1996).

$\frac{1}{3}$ where the domain is $[0, 1]$ for s_1, $[0.1, 1.1]$ for s_2, and $[0.2, 1.2]$ for s_3. The capacity of university U is $q_U = 0.3$. Assume that $p_3^{\bar{g}} = 0.8$, $p_2^{\bar{g}} = 0.5$, and $p_1^{\bar{g}} = 0.2$, so that the proportion of type $\bar{g}$ in the aggregate population is 0.5.

Under the SB policy, there is a unique cutoff score a_{SB} such that any student whose achievement is above a_{SB} is admitted to U. By $q_U = 0.3$, it is easy to find the SB-cutoff: $a_{SB} = 0.8$. Since a school with a larger index sends more students to U and it is correlated with the proportion of the high-SES type $\bar{g}$, the admitted population is biased toward type $\bar{g}$. The SES composition of U in this case is computed as $(0.57, 0.43)$, meaning that 57% of the admitted students are type $\bar{g}$.

Next, consider the top-n policy with $n = 0.3$. Suppose the eligibility requirement and moving costs are such that it is too costly for any students to relocate to any other high school. In this case U admits the top 30% of students from each high school; i.e., the admission cutoffs are $(a_1, a_2, a_3) = (0.7, 0.8, 0.9)$. As a result, the SES composition of the admitted population is $(0.5, 0.5)$ and U is perfectly integrated. Note that the perfect integration is achieved under this scenario because the capacity of U is entirely filled through top-n admissions. It is not always achieved when $q_U > n$, as we will see in Example 2.

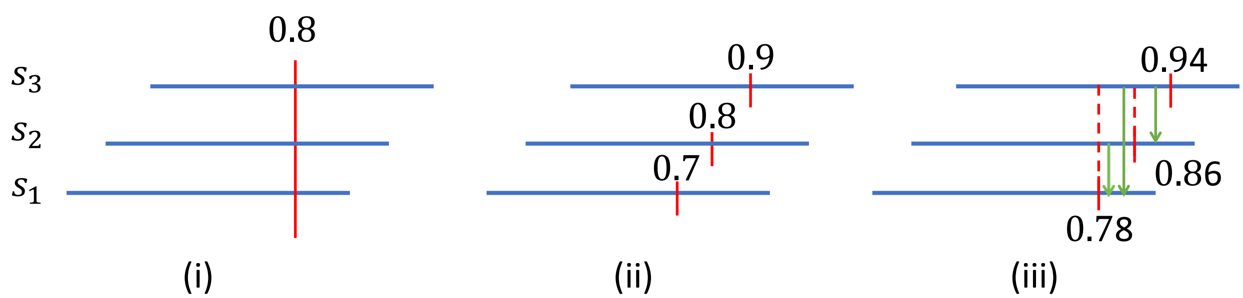

Figure 1: An illustration of (i) the SB policy, (ii) the top-n policy with no student relocation, and (iii) the top-n policy with the relocation of type-$\bar{g}$ students. The red lines are the cutoff scores of schools under each policy. The green arrows represent relocation flows of students in equilibrium.

Under the top-n policy with the same $n = 0.3$, let us now consider a different moving pattern, such that (i) low-SES students still choose not to move, and (ii) a high-SES student who is not admitted to U from her original school is willing to move to any lower-achieving school provided this can lead to her being admitted to U. That is, there are flows of type-$\bar{g}$ students from s_3 to s_2, from s_3 to s_1, and from s_2 to s_1.[9]

[9] Lemma 1 formally shows that, under this moving pattern, the equilibrium cutoff is always monotone,

In this case, we can see that the admission cutoffs (0.7, 0.8, 0.9) of the no-relocation case can no longer be sustained. Given these cutoffs, type-$\bar{g}$ students at s_3 move to s_2 or s_1 if their achievements are in [0.8, 0.9) or [0.7, 0.8), respectively. Once they move out, the population of s_3 goes down, and by the 30% rule, the admission cutoff of s_3 must go up from 0.9. With the flows of students from both s_3 and s_2, s_1's population increases. But since only those who are certain of being admitted to U move to s_1 in equilibrium, they crowd out the original students at s_1 and s_1's cutoff also increases. School s_2 is affected by both forces described above: some students move out to s_1 and others move in from s_3. The market-clearing conditions for the equilibrium cutoffs are:

$$\begin{aligned} 1.2 - a_3 &= 0.3[1 - 0.8(a_3 - a_1)] \\ 1.1 - a_2 + 0.8(a_3 - a_2) &= 0.3[1 + 0.8(a_3 - a_2) - 0.5(a_2 - a_1)] \\ 1 - a_1 + 0.8(a_2 - a_1) + 0.5(a_2 - a_1) &= 0.3[1 + 0.8(a_2 - a_1) + 0.5(a_2 - a_1)] \end{aligned}$$

Each market-clearing condition represents the top-n rule at each school, which requires that the fraction of those admitted from s_i, including migrants, should be exactly $n = 0.3$ of the student population at that school following relocation. The population of migrants between schools is characterized by the cutoffs and the SES distributions. For example, $0.8(a_3 - a_1)$ is the population of students moving out of school s_3. They move to s_2 if their achievement is in $[a_2, a_3)$ and to s_1 if it is in $[a_1, a_2)$. By solving these, the equilibrium cutoff scores when only high-SES students are willing to relocate are approximately $(a_1, a_2, a_3) \simeq (0.78, 0.86, 0.94)$.

In this equilibrium, by strategically moving across schools, any high-SES student with an achievement higher than 0.78 can be admitted to U, irrespective of their original high school. By contrast, low-SES students are admitted only when their achievement is above the cutoff of their original school. The resulting SES composition of the admitted population is $(0.61, 0.39)$, which is even more biased toward the $\bar{g}$ type than it is under the SB policy. We can calculate the MII as the segregation index $\mathcal{I}$ and find that it equals 0.0044 under the top-n policy when only high-SES students move. It is higher than the MII under the SB policy, which equals 0.0017. □

3.2 Top-n policy with no eligibility requirement

We formalize our observation in Example 1 and show that the top-n policy cannot integrate universities unless it controls the relocation of students.

i.e., $a_3 \geq a_2 \geq a_1$ holds in equilibrium.

Proposition 1. *The top-n policy with no eligibility requirement achieves a weakly lower degree of integration than the school-blind policy.*

This result sounds a warning against the use of the top-n policy, which is generally expected to achieve a higher degree of integration than the SB policy. While its message may be surprising, the intuition of this result is straightforward. Recall that condition (iv) of the cost function assumes that the moving cost of a high-SES student is low enough such that they are willing to relocate to any other school just before graduation at T if this means they can be admitted to U instead of u as a result. This is not necessarily the case for the low-SES type; that is, even under the top-n policy with no eligibility requirement, it may still be too costly for a low-SES student to relocate to a school with the lowest cutoff score. In the third case of Example 1 where only high-SES students may relocate, we saw that the lowest cutoff of schools was 0.78, and it was lower than the common cutoff 0.8 under the SB policy. This relationship is a general consequence of market-clearing conditions: we show that, in general, the lowest cutoff score of all high schools under the top-n policy with $e = 0$ is always lower than the common cutoff score under the SB policy. Then, the high-SES type now enjoys the benefit of the lower cutoff score and is able to occupy an even larger share of the seats in U than under the SB policy.

This result is an extension of the "neutrality theorem" shown by Estevan et al. (2018). The neutrality theorem states that when the moving cost is low enough for all students, the set of admitted students under the top-n policy (with no eligibility requirement) becomes identical to the one under the SB policy in equilibrium. By contrast, our setting allows for a realistic structure of cost heterogeneity across SES types, and reveals the possibility that the top-n policy can even harm integration.

3.3 Optimal eligibility requirement

As a solution to the potential problem of the top-n policy presented above, we consider an eligibility requirement e and analyze its optimal level. As we will show below, in the general case with $q_U > n$; that is, when the capacity of U is filled through both top-n and at-large admissions, the most stringent requirement may not always be optimal and the analysis requires a careful examination of different levels of e.

To do so, we classify all possible policies into a finite number of classes, although the eligibility requirement is defined as a continuous variable $e \in [0, T]$. Recall that the cost of relocation $c_g(|i - j|, t)$ is strictly decreasing in the time t of relocation and is strictly increasing in the school difference $|i - j|$. A stricter eligibility requirement e increases the cost by requiring an earlier move at $T-e$, and thus constrains the maximum school difference a student is willing to accept when relocating.[10] Specifically, the maximum school difference equals 2 for SES type g if the requirement e satisfies $c_g(2, T-e) \leq v(U) - v(u)$, equals 1 if $v(U) - v(u) \in [c_g(1, T-e), c_g(2, T-e))$, and equals 0 if $v(U) - v(u) < c_g(1, T-e)$. The moving pattern in equilibrium is characterized by the maximum school difference that each SES type is willing to accept. Given the three high schools and the two SES types, there is a total of six possible moving patterns, as specified in Table 1. We group all requirements that lead to the same moving pattern as the same class of policies because they achieve the same integration level. We name these classes of policies as $\pi_1, \ldots, \pi_6$, as shown in Table 1. We say that policy π_k ($k \in \{1, \ldots, 6\}$) is *feasible* if there exists some eligibility requirement e that induces the moving pattern corresponding to π_k.

policy	π_1	π_2	π_3	π_4	π_5	π_6
students with $\bar{g}$	2	2	2	1	1	0
students with $\underline{g}$	2	1	0	1	0	0

Table 1: Classification of top-n policies

As we increase e, relocation becomes more costly and the maximum school difference weakly decreases for each SES group. Because we assume $c_{\bar{g}}(|i-j|, T) < v(U) - v(u)$ for any $|i-j| \geq 0$ in condition (iv) of the cost function, the top-n policy with no eligibility requirement ($e = 0$) corresponds to one of policies π_1, π_2, or π_3, depending on the cost function of type-$\underline{g}$ students. Condition (v) assumes $c_{\bar{g}}(|i-j|, 0) > v(U) - v(u)$ for any $|i-j| > 0$, thus ensuring that policy π_6 is always feasible when the requirement e is sufficiently close to T. Note that, for any given parameters of the model, at most one of the two policies π_3 and π_4 can be feasible. Therefore, in a given problem, the set of feasible policies can only be one of the following: $\{\pi_1, \pi_2, \pi_3, \pi_5, \pi_6\}$, $\{\pi_1, \pi_2, \pi_4, \pi_5, \pi_6\}$, $\{\pi_1, \pi_2, \pi_5, \pi_6\}$, $\{\pi_2, \pi_3, \pi_5, \pi_6\}$, $\{\pi_2, \pi_4, \pi_5, \pi_6\}$, $\{\pi_2, \pi_5, \pi_6\}$, or $\{\pi_3, \pi_5, \pi_6\}$.

In our framework, we obtain the following clear implication of an eligibility requirement.

Proposition 2. *The optimal top-n policy achieves a weakly higher degree of integration than the school-blind policy.*

We obtain this result without fully characterizing the optimal policy because, in this setting, policy π_6 is always feasible and it always achieves a weakly higher degree of integration

[10]Given an eligibility requirement e, students have the option to move earlier than $T-e$, but they would always choose to move as late as possible to minimize the cost of moving.

than the school-blind policy. Intuitively, policy π_6 admits less (resp. more) students from the highest-achieving school s_3 (resp. the lowest-achieving school s_1) than the SB policy, and increases the proportion of the low-SES type students in the admitted population.

To further characterize the optimal policy among all that are feasible, it is essential to first solve for the ranking of cutoff scores under each possible policy $\pi_1, \ldots, \pi_6$. The following two lemmas summarize the results.

Lemma 1. *In any equilibrium σ under policy $\pi \in \{\pi_1, \pi_3, \pi_6\}$, $a_3(\pi, \sigma) \geq a_2(\pi, \sigma) \geq a_1(\pi, \sigma)$ holds.*

Lemma 2. *In any equilibrium σ under policy $\pi \in \{\pi_2, \pi_4, \pi_5\}$, $a_3(\pi, \sigma) \geq a_2(\pi, \sigma) \geq a_1(\pi, \sigma)$ or $a_2(\pi, \sigma) \geq a_3(\pi, \sigma) \geq a_1(\pi, \sigma)$ holds.*

By exploiting the first-order stochastic dominance relationship between the achievement distributions F_i's, we can show that, in any equilibrium σ under any top-n policy π, $a_2(\pi, \sigma) \geq a_1(\pi, \sigma)$ and $a_3(\pi, \sigma) \geq a_1(\pi, \sigma)$ must hold. However, the relationship between $a_3(\pi, \sigma)$ and $a_2(\pi, \sigma)$ may vary depending on the policy implemented.

Lemma 2 states the non-trivial finding that, under policies π_2, π_4 and π_5, the cutoff ranking may be non-monotone; that is, inconsistent with the ranking of achievement distributions. The intuition behind this result is as follows. Under policies π_2, π_4, and π_5, students of one or both SES types are willing to move only when the school difference is one, implying that those in s_2 may relocate to s_1 while those in s_3 do not relocate to s_1. When the population of s_2 falls following relocation, a_2 is pushed up since top-n admissions are proportional to the high school populations. Then, it is possible that school s_2 loses more students than s_3, and that a_2 becomes higher than a_3 in the equilibrium of policies π_2, π_4, and π_5.

Given the characterization of cutoff scores, we derive the following result that pins down the candidates for the optimal policy.

Proposition 3. *The optimal top-n policy is policy π_6 when policy π_4 is not feasible.*

Note that the equilibrium of policy π_1 achieves the same outcome as the SB policy by the neutrality theorem of Estevan et al. (2018), and that Proposition 1 shows that policy π_1 achieves a weakly higher degree of integration than policies π_2 and π_3. Then, for Proposition 3, it suffices to show that policy π_6 achieves a weakly higher degree of integration than policy π_5. The key to the comparison between policies π_5 and π_6 is that low-SES students do not move under either policy. Since high-SES students may move across high schools under policy π_5, this induces a higher equilibrium cutoff score for every school than under policy

π_6, thereby reducing admissions of low-SES students who can only be admitted to U from their original schools.

Proposition 3 leaves open the possibility that policy π_6 may not be optimal when policy π_4 is feasible. It is easy to see that when the capacity of U is entirely filled through top-n admissions, i.e., $n = q_U$, policy π_6 admits the representative population of students ($p_U^{\bar{g}} = p^{\bar{g}}$) and is always optimal. However, when $q_U > n$, the admitted population under policy π_6 is not always representative, and is biased toward the high-SES type ($p_U^{\bar{g}} > p^{\bar{g}}$) since the $q_U - n$ students admitted at large are mostly from high-achieving schools. If, under policy π_4, students from low-achieving schools occupy a higher share of the $q_U - n$ population admitted at large than under π_6, then policy π_4 can lead to a higher degree of integration than policy π_6. The following example demonstrates that this scenario may occur in equilibrium.

Example 2. Suppose each high school s_i has the same measure of students $q_i = \frac{1}{3}$. At each high school, student achievement a is uniformly distributed with density $\frac{1}{3}$ and the domain is $[0, 1]$ for s_1, $[0.2, 1.2]$ for s_2, and $[0.22125, 1.22125]$ for s_3. The capacity of U is $q_U = 0.203$. Consider the top-n policy with $n = 0.2$. Assume that $p_3^{\bar{g}} = 1$ and $p_2^{\bar{g}} = p_1^{\bar{g}} = 0$.

Under each policy, let a_i^N be the lowest achievement of students from high school s_i admitted through top-n admissions. Let a^L be the lowest achievement of students (from any high school) admitted through at-large admissions. Then, each school's cutoff score a_i is found by $a_i = \min\{a_i^N, a^L\}$.

Under policy π_6, there is no relocation across high schools and the cutoff scores are given by $(a_1^N(\pi_6), a_2^N(\pi_6), a_3^N(\pi_6), a^L(\pi_6)) = (0.8, 1, 1.02125, 1.01225)$. The SES composition vector of students admitted to U is $p_U(\pi_6) = \frac{1}{0.203}\frac{1}{3}(0.209, 0.4) \simeq (0.343, 0.657)$. Since the SES composition in the aggregate population is $p = (\frac{1}{3}, \frac{2}{3})$, p_U is biased toward the $\bar{g}$ type and hence, $\mathcal{I}(\pi_6) \simeq 2.5 \cdot 10^{-5} > 0$.

Under policy π_4, it is easy to check that $a_3 = a_2 = a^L$ holds in the unique equilibrium. The market-clearing conditions are:

$$\begin{aligned} 1.22125 - a_3^N &= 0.2 \\ 1.2 - a_2^N &= 0.2[1 - (a^L - a_1^N)] \\ a_1^N &= 0.8[1 + (a^L - a_1^N)] \\ \frac{1}{3}(a_3^N - a^L + a_2^N - a^L) &= 0.003 \end{aligned}$$

By solving these, we obtain the cutoff scores

$$(a_1^N(\pi_4), a_2^N(\pi_4), a_3^N(\pi_4), a^L(\pi_4)) = (0.897, 1.02425, 1.02125, 1.01825).$$

The SES composition vector of students admitted to U is $p_U(\pi_4) = \frac{1}{0.203}\frac{1}{3}(0.203, 0.406) = (\frac{1}{3}, \frac{2}{3})$. By $p_U = p$, policy π_4 integrates U perfectly and hence, $\mathcal{I}(\pi_4) = 0$. $\square$

In this example, the unique equilibrium of policy π_4 integrates U perfectly, while policy π_6 does not. The logic behind this observation is that when students are willing to relocate only when the school difference is one under policy π_4, the population of those students admitted to U from s_3 can be smaller than that under policy π_6. To see this, notice that under policy π_4, students at s_2 can exploit the low cutoff score of s_1 by moving to s_1. This relocation reduces the total population of s_2 and hence raises its cutoff score for top-n admissions to $a_2^N(\pi_4) = 1.02425$, compared with the cutoff under π_6, which is $a_2^N(\pi_6) = 1$. As a result, students from s_2 occupy a larger share of at-large admissions under policy π_4 than under policy π_6, and the population of students admitted from s_3 under policy π_4 (which is $\frac{1}{3} \cdot 0.203$) becomes smaller than that under policy π_6 (which is $\frac{1}{3} \cdot 0.209$). Combined with $(p_1^{\bar{g}}, p_2^{\bar{g}}, p_3^{\bar{g}}) = (1, 0, 0)$, fewer $\bar{g}$-type students are admitted to U under policy π_4 than under policy π_6, correcting the bias of policy π_6 and achieving perfect integration in this case.[11]

To further understand how policy π_4 compares with policy π_6 in different scenarios, the next propositions provide the conditions under which either policy π_4 or π_6 is optimal in terms of integration. Let $\underline{a}_{i,j}(\pi, \sigma) := \min\{a_i(\pi, \sigma), a_j(\pi, \sigma)\}$ be the minimum of the cutoff scores of two schools s_i and s_j in equilibrium σ under policy π.[12] The optimal policy is mainly determined by how $\underline{a}_{2,3}$ under policy π_4 compares with a_3 under policy π_6.

Proposition 4. *Consider a problem where policy π_4 is feasible. Policy π_6 is optimal when $\underline{a}_{2,3}(\pi_4, \sigma) \leq a_3(\pi_6)$ holds in any equilibrium σ under policy π_4.*

Proposition 5. *Consider a problem where $n < q_U$ and policy π_4 is feasible. When $\underline{a}_{2,3}(\pi_4, \sigma) > a_3(\pi_6)$ holds in some equilibrium σ under policy π_4, there exists an interval $(x, y) \subset [p_1^{\bar{g}}, p_3^{\bar{g}}]$ of $p_2^{\bar{g}}$ such that $\mathcal{I}(\pi_4, \sigma) < \mathcal{I}(\pi_6)$. That is, for any $p_2^{\bar{g}} \in (x, y)$, policy π_6 is not optimal.*

The relationship between $\underline{a}_{2,3}(\pi_4, \sigma)$ and $a_3(\pi_6)$ is critical because it determines which policy sends more students originally from school s_3 to the target university U. Since students are willing to move only when the school difference is one under policy π_4 (irrespective of their SES type), the admitted population of students who are originally enrolled in s_3 is computed as $p_3^{\bar{g}} q_3[1 - F_3(\underline{a}_{2,3}(\pi_4, \sigma))]$. On the other hand, since there is no relocation under policy π_6, the admitted population of students from school s_3 is given by $p_3^{\bar{g}} q_3[1 - F_3(a_3(\pi_6))]$.

[11] $(p_1^{\bar{g}}, p_2^{\bar{g}}, p_3^{\bar{g}}) = (1, 0, 0)$ is only used to achieve perfect integration under policy π_4 in this example. As seen in Proposition 5, this assumption is not necessary for showing the suboptimality of policy π_6.

[12] Under policy π_4, there may exist multiple equilibria. The most relevant case is when a_2 is strictly higher than a_3 and students in s_2 with $a \in [a_3, a_2)$ are indifferent between moving to s_1 and moving to s_3.

In addition, applying the same logic used to analyze policy π_5 in Proposition 3, we know that $a_1(\pi_4, \sigma)$ is always higher than $a_1(\pi_6)$. This implies that policy π_4 always sends fewer students from school s_1 than policy π_6. Combined with the population of those originally from s_3, under the condition $\underline{a}_{2,3}(\pi_4, \sigma) \leq a_3(\pi_6)$ of Proposition 4, policy π_4 sends more students originally from higher-achieving schools (only s_1, or both s_1 and s_2), thereby achieving a lower degree of integration than π_6. Under the condition $\underline{a}_{2,3}(\pi_4, \sigma) > a_3(\pi_6)$ of Proposition 5, the degree of integration depends on the SES distribution p_2^g of school s_2, because policy π_4 sends fewer students originally from s_1 and s_3 but more students originally from s_2 than does π_6. When $p_2^{\bar{g}}$ is low enough, policy π_4 sends strictly fewer type-$\bar{g}$ students to U than does π_6, and hence the equilibrium of policy π_4 can achieve a higher degree of integration than π_6 for some value of $p_2^{\bar{g}} \in [p_1^{\bar{g}}, p_3^{\bar{g}}]$.

3.4 Uniform distribution

The conditions for the optimal top-n policy in Propositions 4 and 5 are characterized in terms of the cutoff scores in equilibrium. This section focuses on a model with a uniform distribution of achievement, and illustrates, in terms of model parameters, when and how policy π_4 can achieve a higher level of integration than policy π_6.

Consider a model with $q_U > n$. Suppose that originally each high school s_i has the same measure of students $q_i = \frac{1}{3}$. At each high school, a student's achievement a is uniformly distributed with density $\frac{1}{3}$ and the domain is $[0, 1]$ for s_1, $[d_1, 1 + d_1]$ for s_2, and $[d_1 + d_2, 1 + d_1 + d_2]$ for s_3, where $d_1 > 0$ and $d_2 > 0$.

Proposition 6. *In the model specified in Section 3.4, there exists an equilibrium σ under policy π_4 such that $\underline{a}_{2,3}(\pi_4, \sigma) > a_3(\pi_6)$ if $q_U - n \in (\max\{0, -\frac{n}{6(1-n)}d_1 + \frac{1}{3}d_2\}, \frac{2}{3}d_1 + \frac{1}{3}d_2)$.*

This result tells us that, for any $d_1 > 0$ and $d_2 > 0$, there is a range of $q_U - n$ such that policy π_6 is not optimal. The intuition for the range of $q_U - n$ is as follows. The upper-bound condition $q_U - n < \frac{2}{3}d_1 + \frac{1}{3}d_2$ is used to rule out the cases where a large fraction of students can be admitted at large and all high schools have identical cutoff scores under either policy π_6 or π_4. The lower-bound condition $q_U - n > \max\{0, -\frac{n}{6(1-n)}d_1 + \frac{1}{3}d_2\}$ is satisfied when d_1 is relatively larger compared with d_2. In this case, because of the large difference between the achievement distributions of s_2 and s_1, many students at s_2 can relocate to s_1 under policy π_4, which pushes up the cutoff score of s_2. In an equilibrium σ where all students at s_2 with achievement $a \in [a_1(\pi_4, \sigma), a_2(\pi_4, \sigma))$ relocate to s_1, we can prove that $\underline{a}_{2,3}(\pi_4, \sigma) > a_3(\pi_6)$ holds under this lower-bound condition.

We illustrate the implications of Propositions 5 and 6 with a numerical example, in which MII is used to indicate the degree of segregation $\mathcal{I}$.[13] In Figure 2, we fix $(d_1, d_2, n, q_U, p_1^{\bar{g}}, p_3^{\bar{g}})$ and change the value of $p_2^{\bar{g}}$ in the horizontal axis. The parameter set is chosen to satisfy the condition in Proposition 6 and thus the premise of Proposition 5. As implied by Proposition 5, we indeed find a range of $p_2^{\bar{g}}$ such that policy π_4 achieves a higher degree of integration (a smaller $\mathcal{I}$) than policy π_6.

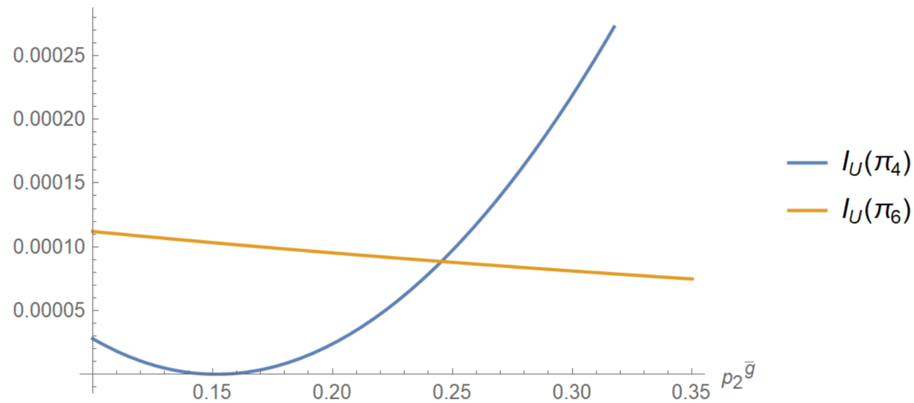

Figure 2: Degree of segregation (MII) and $p_2^{\bar{g}}$ under policies π_4 and π_6 ($d_1 = 0.3$, $d_2 = 0.04$, $n = 0.6$, $q_U = 0.61$, $p_1^{\bar{g}} = 0.1$, $p_3^{\bar{g}} = 0.9$).

4 Discussions

4.1 Implication for a general setting

While we analyzed a model with three high schools and two SES types, its main implications hold more generally in the following senses.

Implication of Proposition 1. The message of Proposition 1 is that the top-n policy with no eligibility requirement always fares (weakly) worse than the SB policy in achieving university integration. It is indeed straightforward to extend this result to a setting where there are four or more high schools. The key assumption is that the relocation cost of type $\bar{g}$ at time T is low enough such that high-SES students are willing to move to any high school in order to improve their admission outcome when there is no eligibility requirement.

[13] In this example, under policy π_4, we consider an equilibrium σ where all students at s_2 with achievement $a \in [a_1(\pi_4, \sigma), a_2(\pi_4, \sigma))$ relocate to s_1, even when $a_3(\pi_4, \sigma) < a_2(\pi_4, \sigma)$ such that they are indifferent between moving to s_1 and moving to s_3.

Note that this is a natural extension of condition (iv) of our cost function. Therefore, the importance of an eligibility requirement is not specific to our three-school environment.[14]

Implication of Example 2. The other main message of our theoretical results is that the most stringent eligibility requirement is not necessarily optimal. It is easy to see that Example 2 can be "embedded" in a setting with any number of high schools and SES types. That is, for any number of high schools and SES types, we can find a set of parameters such that the most stringent eligibility requirement is not optimal. Although we do not provide the characterization of the optimal top-n policy in a general setting, the message that a careful choice of an eligibility requirement is important is generally applicable.

4.2 Integration of high schools

Our results imply that it is sometimes preferable to permit student relocation (by choosing an intermediate level of e) in order to achieve higher university integration. From another perspective, Estevan et al. (2018) argue that relocation incentives can be used as an instrument to integrate high schools rather than universities. This is based on the idea that if students from high-achieving schools relocate to lower-achieving schools, this renders the SES distributions of high schools more uniform.[15] Although students tend to relocate as late as possible, high schools can be integrated for a certain period of time before graduation if students are incentivized to relocate early to meet some eligibility requirement.

The unbiased mixing theorem of Estevan et al. (2018) shows that any relocation of students leads to a (weakly) higher degree of integration of high schools. This theorem applies to policies π_4 and π_6 in our model because it requires that any relocation is "unbiased"; i.e., that the population that moves out of a high school has the same SES distribution as the original population at that high school. Since some students relocate under policy π_4 while no students relocate under policy π_6, the unbiased mixing theorem implies that policy π_4 always achieves a (weakly) higher degree of high-school integration than policy π_6. This means that, in addition to the cases described in Proposition 5, there is an added rationale for policy π_4 if the policymaker also cares about the integration of the high school sector.

[14]While we can allow any number of high schools for this result, it is not straightforward to relax the assumption of two SES types. We thus leave the question of whether this result continues to hold with three or more SES types for future research.

[15]Estevan et al. (2018) consider only the top-n policy with no eligibility requirement. If it is more costly to move earlier than to move later, students would only want to relocate close to graduation. This indicates another benefit of imposing an eligibility requirement: without such a requirement, the effect of the top-n policy on high-school integration can also be rather limited.

5 Conclusion

In this paper, we offer a simple model crafted to analyze the effects of widely used top-n policies on the diversity of university campuses. Our model has realistic features such as the strategic relocation decisions of students, the requirement for a minimum amount of time to be spent in a high school for students to be eligible for top-n admissions, and the differentiation of students' relocation costs based on their SES.

The main takeaway from our paper is that, in order to achieve integration outcomes, top-n policies need to be accompanied by carefully chosen eligibility requirements. Without any eligibility requirement, the top-n policy may result in worse outcomes than when it is not used at all. However, it can achieve a higher degree of integration than the school-blind policy when an appropriate level of requirement is adopted. Surprisingly, stricter eligibility requirements do not always perform better than less strict ones, meaning that the optimal level of eligibility requirement needs to be carefully chosen. We also note that our main implications continue to hold in more general settings with a greater number of high schools.

A more general message of this paper is that policies formulated with the important aim of desegregation may have unintended consequences, and need to be carefully designed incorporating a rigorous economics analysis. We also contribute to the debate on the effectiveness of "race-based affirmative action policies" versus "color-blind affirmative action policies", since we analyze strategic responses under the top-n policy, which is a prevailing example of a color-blind policy. We hope that, with more theoretical and empirical work on this important topic, researchers will not only evaluate current and alternative policies, but will also guide policymakers toward implementing more effective strategies and programs.

References

Abdulkadiroğlu, A. and T. Sönmez (2003): "School Choice: A Mechanism Design Approach," *American Economic Review*, 93, 729–747.

Andrews, R. J., V. Ranchhod, and V. Sathy (2010): "Estimating the responsiveness of college applications to the likelihood of acceptance and financial assistance: Evidence from Texas," *Economics of Education Review*, 29, 104–115.

Aygün, O. and B. Turhan (2020): "Dynamic reserves in matching markets," *Journal of Economic Theory*, 188, 105069.

Black, S. E., J. T. Denning, and J. Rothstein (2020): "Winners and losers? the effect of gaining and losing access to selective colleges on education and labor market outcomes," Tech. rep., National Bureau of Economic Research.

Bleemer, Z. (2021): "Top percent policies and the return to postsecondary selectivity," Unpublished manuscript.

Cortes, K. and D. Klasik (2020): "Uniform Admissions, Unequal Access: Did the Top 10% Plan Increase Access to Selective Flagship Institutions?" Tech. rep., National Bureau of Economic Research.

Cortes, K. E. (2010): "Do bans on affirmative action hurt minority students? Evidence from the Texas Top 10% Plan," *Economics of Education Review*, 29, 1110–1124.

Cortes, K. E. and A. I. Friedson (2014): "Ranking up by moving out: the effect of the Texas top 10% plan on property values," *National Tax Journal*, 67, 51–76.

Cullen, J. B., M. C. Long, and R. Reback (2013): "Jockeying for Position: Strategic High School Choice under Texas' Top Ten Percent Plan," *Journal of Public Economics*, 97, 32–48.

Daugherty, L., P. Martorell, and I. McFarlin (2014): "Percent plans, automatic admissions, and college outcomes," *IZA Journal of Labor Economics*, 3, 1–29.

Dur, U., S. D. Kominers, P. A. Pathak, and T. Sönmez (2018): "Reserve design: Unintended consequences and the demise of Boston's walk zones," *Journal of Political Economy*, 126, 2457–2479.

Dur, U., P. A. Pathak, and T. Sönmez (2020): "Explicit vs. statistical targeting in affirmative action: Theory and evidence from Chicago's exam schools," *Journal of Economic Theory*, 187, 104996.

Ehlers, L., I. E. Hafalir, M. B. Yenmez, and M. A. Yildirim (2014): "School choice with controlled choice constraints: Hard bounds versus soft bounds," *Journal of Economic theory*, 153, 648–683.

Ellison, G. and P. A. Pathak (2021): "The Efficiency of Race-Neutral Alternatives to Race-Based Affirmative Action: Evidence from Chicago's Exam Schools," *American Economic Review*, 111, 943–975.

Estevan, F., T. Gall, P. Legros, and A. F. Newman (2018): "The Top-Ten Way to Integrate High Schools," Unpublished manuscript.

Fletcher, J. M. and A. Mayer (2014): "Tracing the effects of guaranteed admission through the college process: evidence from a policy discontinuity in the Texas 10% plan," *Contemporary economic policy*, 32, 169–186.

Fragiadakis, D. and P. Troyan (2017): "Improving matching under hard distributional constraints," *Theoretical Economics*, 12, 863–908.

Fryer Jr, R. G., G. Loury, and T. Yuret (2003): "Color-blind affirmative action," National Bureau of Economic Research Cambridge, Mass., USA.

Hafalir, I. E., M. B. Yenmez, and M. A. Yildirim (2013): "Effective affirmative action in school choice," *Theoretical Economics*, 8, 325–363.

Harris, A. L. and M. Tienda (2012): "Hispanics in higher education and the Texas top 10% law," *Race and social problems*, 4, 57–67.

James, D. R. and K. E. Taeuber (1985): "Measures of Segregation," *Social Methodology*, 15, 1–32.

Kain, J. F., D. M. O'Brien, and P. A. Jargowsky (2005): "Hopwood and the Top 10 Percent Law: How They Have Affected the College Enrollment Decisions of Texas High School Graduates," Unpublished manuscript, the University of Texas at Dallas.

Kapor, A. (2020): "Distributional effects of race-blind affirmative action," Unpublished manuscript.

Kojima, F. (2012): "School choice: Impossibilities for affirmative action," *Games and Economic Behavior*, 75, 685–693.

Kominers, S. D. and T. Sönmez (2016): "Matching with slot-specific priorities: Theory," *Theoretical Economics*, 11, 683–710.

Kremer, M. and E. Maskin (1996): "Wage Inequality and Segregation by Skill," NBER Working Paper 5718.

Long, M. C. (2007): "Affirmative action and its alternatives in public universities: What do we know?" *Public Administration Review*, 67, 315–330.

Long, M. C. and N. A. Bateman (2020): "Long-run changes in underrepresentation after affirmative action bans in public universities," *Educational Evaluation and Policy Analysis*, 42, 188–207.

Long, M. C., V. Saenz, and M. Tienda (2010): "Policy transparency and college enrollment: did the Texas top ten percent law broaden access to the public flagships?" *The ANNALS of the American Academy of Political and Social Science*, 627, 82–105.

Long, M. C. and M. Tienda (2010): "Changes in Texas universities' applicant pools after the Hopwood decision," *Social Science Research*, 39, 48–66.

Montejano, D. (2001): *Access to the University of Texas at Austin and the ten percent plan: A three-year assessment*, University of Texas Austin, TX.

Niu, S. X. and M. Tienda (2010): "The impact of the Texas top ten percent law on college enrollment: A regression discontinuity approach," *Journal of Policy Analysis and Management*, 29, 84–110.

Niu, S. X., M. Tienda, and K. Cortes (2006): "College selectivity and the Texas top 10% law," *Economics of Education Review*, 25, 259–272.

Theil, H. and A. J. Finizza (1971): "A Note on the Measurement of Racial Integration of Schools by Means of Informational Concepts," *Journal of Mathematical Sociology*, 1, 187–194.

Tomoeda, K. (2018): "Finding a stable matching under type-specific minimum quotas," *Journal of Economic Theory*, 176, 81–117.

Westkamp, A. (2013): "An analysis of the German university admissions system," *Economic Theory*, 53, 561–589.

Appendix A Existence of an equilibrium under the top-n policy

First, we can see that under the top-n policy with any eligibility requirement $e \in [0, T]$, any equilibrium has cutoff scores (a_1, a_2, a_3), where a_i is the cutoff score for high school s_i. The top-n policy admits n students through top-n admissions and $q_U - n$ students through at-large admissions. If a student with achievement a is admitted to U from high school s_i through top-n admissions, any student with $a' > a$ from the same high school is also admitted through top-n admissions. If a student with achievement a is admitted to U through at-large admissions, any student with $a' > a$ is also admitted from any high school. Therefore, the top-n cutoff score a_i^N is defined as the lowest achievement of admitted students from high school s_i through top-n admissions. Similarly, the at-large cutoff score a^L can be defined as the lowest achievement of admitted students (from any high school) through at-large admissions. Since students may or may not be admitted through at-large admissions from a given high school, the cutoff score a_i is defined as $a_i := \min\{a_i^N, a^L\}$.

To prove the existence of an equilibrium under the top-n policy with any eligibility requirement $e \in [0, T]$, let us define the following for a student with (a, g, s_i) given a vector of cutoff scores $\mathbf{a} = (a_1^N, a_2^N, a_3^N, a^L)$:

$$M_{s_i}^g(a, e, \mathbf{a}) = \left\{ s_j \in \{s_1, s_2, s_3\} \setminus \{s_i\} \middle| c_g(|i-j|, T-e) \leq v(U) - v(u) \text{ and } a \in [a_j, a_i) \right\}$$

$$h_{s_i}^g(a, e, \mathbf{a}) = \begin{cases} s_i & \text{if } a \geq a_i \\ s_j & \text{if } s_j \in M_{s_i}^g(a, e, \mathbf{a}) \text{ and } c_g(|i-j|, T-e) \leq c_g(|i-k|, T-e) \text{ for any } s_k \in M_{s_i}^g(a, e, \mathbf{a}) \\ \emptyset & \text{if } a < a_i \text{ and } M_{s_i}^g(a, e, \mathbf{a}) = \{\emptyset\} \end{cases}$$

In words, $M_{s_i}^g(a, e, \mathbf{a})$ is the set of high schools that a student with (a, g, s_i) has an incentive to relocate to given the eligibility requirement e and cutoff scores $\mathbf{a}$. $h_{s_i}^g(a, e, \mathbf{a})$ is the high school from which this student is admitted to U in the equilibrium with cutoff scores $\mathbf{a}$ (which takes $\emptyset$ when this student cannot be admitted to U).

In any equilibrium, the cutoff scores $\mathbf{a} = (a_1^N, a_2^N, a_3^N, a^L)$ satisfy the following four market-clearing conditions. The market-clearing condition for each a_i^N is

$$\begin{aligned}
&q_i[1-F_i(a_i^N)] + \sum_{j\in\{1,2,3\}\setminus\{i\}} q_j \sum_{g\in\{\bar{g},\underline{g}\}} p_j^g \int_{a:s_i=h_{s_j}^g(a,e,\mathbf{a})} f_j(a)da \\
=&n\Big[q_i[1-F_i(a_i^N)] + \sum_{j\in\{1,2,3\}\setminus\{i\}} q_j \sum_{g\in\{\bar{g},\underline{g}\}} p_j^g \int_{a:s_i=h_{s_j}^g(a,e,\mathbf{a})} f_j(a)da \\
&+ \max\Big\{q_i[F_i(a_i^N)-F_i(a^L)],0\Big\} + q_i \sum_{g\in\{\bar{g},\underline{g}\}} p_i^g \int_{a:\emptyset=h_{s_i}^g(a,e,\mathbf{a})} f_i(a)da\Big].
\end{aligned}$$

The market-clearing condition for a^L is

$$\sum_{i=1}^{3} \max\Big\{q_i[F_i(a_i^N)-F_i(a^L)],0\Big\} = q_U - n.$$

Then, we can define the following excess demand functions:

$$\begin{aligned}
z_i(\mathbf{a}) =&(1-n)\Big[q_i[1-F_i(a_i^N)] + \sum_{j\in\{1,2,3\}\setminus\{i\}} q_j \sum_{g\in\{\bar{g},\underline{g}\}} p_j^g \int_{a:s_i=h_{s_j}^g(a,e,\mathbf{a})} f_j(a)da\Big] \\
&- n\Big[\max\Big\{q_i[F_i(a_i^N)-F_i(a^L)],0\Big\} + q_i \sum_{g\in\{\bar{g},\underline{g}\}} p_i^g \int_{a:\emptyset=h_{s_i}^g(a,e,\mathbf{a})} f_i(a)da\Big] \text{ for } i=1,2,3, \\
z_4(\mathbf{a}) =&\sum_{i=1}^{3} \max\Big\{q_i[F_i(a_i^N)-F_i(a^L)],0\Big\} - (q_U-n).
\end{aligned}$$

Define a map $B:[0,\bar{a}]^4 \to [0,\bar{a}]^4$ such that

$$\begin{aligned}
B_i(\mathbf{a}) =& \max\big\{\min\{a_i^N + z_i(\mathbf{a}), \bar{a}\}, 0\big\} \text{ for } i=1,2,3, \\
B_4(\mathbf{a}) =& \max\big\{\min\{a^L + z_4(\mathbf{a}), \bar{a}\}, 0\big\}.
\end{aligned}$$

Since $z_i(\cdot)$ is continuous by the continuity of $F_i(\cdot)$, $B(\cdot)$ is continuous. Then by Brouwer's theorem, $B(\cdot)$ has a fixed point $\mathbf{a}^* = (a_1^{N*}, a_2^{N*}, a_3^{N*}, a^{L*})$. We can show that $z_i(\mathbf{a}^*) = 0$ is satisfied for all $i \in \{1,\ldots,4\}$, and hence $\mathbf{a}^*$ is a vector of equilibrium cutoff scores.

By $B(\mathbf{a}^*) = \mathbf{a}^*$, it is clear that $z_i(\mathbf{a}^*) = 0$ holds for all $i \in \{1,\ldots,4\}$ if $\mathbf{a}^* \in (0,\bar{a})^4$. If $a_i^{N*} = \bar{a}$ happens, $z_i(\mathbf{a}^*) \leq 0$. In this case, $z_i(\mathbf{a}^*) = 0$ is necessary because $z_i(\mathbf{a}) < 0$ would imply $B_i(\mathbf{a}^*) < a_i^{N*}$, which is a contradiction. If $a^{L*} = \bar{a}$, $q_U - n = 0$ must hold and we have $z_4(\mathbf{a}^*) = 0$. Next, $a_i^{N*} = 0$ should not happen because $a_i^{N*} = 0$ implies $z_i(\mathbf{a}^*) > 0$ by $n < 1$, and it is a contradiction to $B_i(\mathbf{a}^*) = a_i^{N*}$. $a^{L*} = 0$ should not happen either because $a^{L*} = 0$ would imply $z_4(\mathbf{a}^*) > 0$ by $q_U < 1$, and this is a contradiction to $B_4(\mathbf{a}^*) = a^{L*}$.

Appendix B Omitted proofs

Appendix B.1 Proof of Proposition 1

By assumption (iv) of the cost function, students with SES type $\bar{g}$ have the incentive to move to any high schools at time T if they can be admitted to U by doing so. On the other hand, the maximum difference of school indices that those with SES type $\underline{g}$ are willing to accept is either 2, 1 or 0.

Consider these three scenarios under the top-n policy with no eligibility requirement ($e = 0$). Take any equilibrium σ of the top-n policy with $e = 0$. When the low-SES type $\underline{g}$ is willing to move to any high schools at time T, the neutrality theorem of Estevan et al. (2018) implies that the top-n policy with no eligibility requirement achieves the same level of integration as the SB policy.

When the low-SES type $\underline{g}$ is willing to move by one school at time T, the sum of market-clearing conditions for three high schools is

$$\begin{aligned}&q_3F_3(\underline{a}(0,\sigma)) + q_2F_2(\underline{a}(0,\sigma)) + q_1F_1(\underline{a}(0,\sigma))\\&+ \mathbb{1}_{\{\underline{a}_{2,3}(0,\sigma)\geq a_1(0,\sigma)\}}p_3^{\underline{g}}q_3[F_3(\{\underline{a}_{2,3}(0,\sigma)) - F_3(a_1(0,\sigma))]\\&+ \mathbb{1}_{\{\underline{a}_{1,2}(0,\sigma)\geq a_3(0,\sigma)\}}p_1^{\underline{g}}q_1[F_1(\{\underline{a}_{1,2}(0,\sigma)) - F_1(a_3(0,\sigma))]\\=&1 - q_U\end{aligned}$$

where $\underline{a}(0,\sigma) := \min\{a_1(0,\sigma), a_2(0,\sigma), a_3(0,\sigma)\}$. Similarly, when the low-SES type $\underline{g}$ is not willing to move to any other school at time T, the sum of market-clearing conditions for three high schools is

$$\begin{aligned}&q_3F_3(\underline{a}(0,\sigma)) + q_2F_2(\underline{a}(0,\sigma)) + q_1F_1(\underline{a}(0,\sigma)) +\\&p_3^{\underline{g}}q_3[F_3(a_3(0,\sigma)) - F_3(\underline{a}(0,\sigma))] + p_2^{\underline{g}}q_2[F_2(a_2(0,\sigma)) - F_2(\underline{a}(0,\sigma))] + p_1^{\underline{g}}q_1[F_1(a_1(0,\sigma)) - F_1(\underline{a}(0,\sigma))]\\=&1 - q_U.\end{aligned}$$

In either scenario, we have

$$q_3F_3(\underline{a}(0,\sigma)) + q_2F_2(\underline{a}(0,\sigma)) + q_1F_1(\underline{a}(0,\sigma)) \leq 1 - q_U = q_3F_3(a_{SB}) + q_2F_2(a_{SB}) + q_1F_1(a_{SB}),$$

where a_{SB} is the cutoff score under the SB policy. Then $\underline{a}(0,\sigma) \leq a_{SB}$ holds. This implies that the population of the high-SES type $\bar{g}$ at U under the top-n policy with $e = 0$ is larger than that under the SB policy:

$$\begin{aligned}&p_3^{\bar{g}}q_3[1 - F_3(\underline{a}(0,\sigma))] + p_2^{\bar{g}}q_2[1 - F_2(\underline{a}(0,\sigma))] + p_1^{\bar{g}}q_1[1 - F_1(\underline{a}(0,\sigma))]\\\geq&p_3^{\bar{g}}q_3[1 - F_3(a_{SB})] + p_2^{\bar{g}}q_2[1 - F_2(a_{SB})] + p_1^{\bar{g}}q_1[1 - F_1(a_{SB})].\end{aligned}$$

Note that the share of SES type $\bar{g}$ at U under the SB policy is larger than the population average $p^{\bar{g}}$:

$$\frac{p_3^{\bar{g}} q_3[1-F_3(a_{SB})] + p_2^{\bar{g}} q_2[1-F_2(a_{SB})] + p_1^{\bar{g}} q_1[1-F_1(a_{SB})]}{q_3[1-F_3(a_{SB})] + q_2[1-F_2(a_{SB})] + q_1[1-F_1(a_{SB})]} \geq \frac{p_3^{\bar{g}} q_3 + p_2^{\bar{g}} q_2 + p_1^{\bar{g}} q_1}{q_3+q_2+q_1} = p^{\bar{g}}.$$

Therefore, the share of $\bar{g}$ at two universities under the top-n policy with $e=0$ diverges from the SB policy, and $\mathcal{I}(0,\sigma) \geq \mathcal{I}(SB)$ holds.

Appendix B.2 Proof of Proposition 2

By the neutrality theorem of Estevan et al. (2018), the equilibrium outcome of π_1 is equivalent to the outcome of the SB policy. Then, it suffices to show that π_6 achieves weakly higher integration than π_1.

First, we can see that the share of $\bar{g}$ students at U under π_6 is greater than the population average of type $\bar{g}$. The population of $\bar{g}$ students admitted to U under policy π_6 is

$$p_3^{\bar{g}} q_3[1-F_3(a_3(\pi_6))] + p_2^{\bar{g}} q_2[1-F_2(a_2(\pi_6))] + p_1^{\bar{g}} q_1[1-F_1(a_1(\pi_6))].$$

Under π_6, the top n fraction of students who are originally at each school are admitted, and the rest of $q_U - n$ students are admitted according to their achievement. Then, combined with the first-order stochastic dominance relationship of F_i's, $1-F_3(a_3(\pi_6)) \geq 1-F_2(a_2(\pi_6)) \geq 1-F_1(a_1(\pi_6))$ holds. By $p_3^{\bar{g}} \geq p_2^{\bar{g}} \geq p_1^{\bar{g}}$, the share of $\bar{g}$ students at U under π_6 is greater than the population average $p^{\bar{g}}$:

$$\frac{p_3^{\bar{g}} q_3[1-F_3(a_3(\pi_6))] + p_2^{\bar{g}} q_2[1-F_2(a_2(\pi_6))] + p_1^{\bar{g}} q_1[1-F_1(a_1(\pi_6))]}{q_3[1-F_3(a_3(\pi_6))] + q_2[1-F_2(a_2(\pi_6))] + q_1[1-F_1(a_1(\pi_6))]} \geq \frac{p_3^{\bar{g}} q_3 + p_2^{\bar{g}} q_2 + p_1^{\bar{g}} q_1}{q_3+q_2+q_1} = p^{\bar{g}}.$$

Next, by the sum of three market-clearing conditions for high schools,

$$1-q_U = q_3F_3(a_1(\pi_1)) + q_2F_2(a_1(\pi_1)) + q_1F_1(a_1(\pi_1)) = q_3F_3(a_3(\pi_6)) + q_2F_2(a_2(\pi_6)) + q_1F_1(a_1(\pi_6)).$$

Together with Lemma 1, this implies $a_1(\pi_6) \leq a_1(\pi_1) \leq a_3(\pi_6)$. Then, by $p_3^{\bar{g}} \geq p_2^{\bar{g}} \geq p_1^{\bar{g}}$, we have

$$\begin{aligned} &\frac{p_3^{\bar{g}} q_3[1-F_3(a_1(\pi_1))] + p_2^{\bar{g}} q_2[1-F_2(a_1(\pi_1))] + p_1^{\bar{g}} q_1[1-F_1(a_1(\pi_1))]}{q_U} \\ \geq &\frac{p_3^{\bar{g}} q_3[1-F_3(a_3(\pi_6))] + p_2^{\bar{g}} q_2[1-F_2(a_2(\pi_6))] + p_1^{\bar{g}} q_1[1-F_1(a_1(\pi_6))]}{q_U}, \end{aligned}$$

meaning that the share of the $\bar{g}$ students admitted at U under π_1 is larger than under π_6 (which is larger than the population average $p^{\bar{g}}$). Therefore, the integration of U under π_1 is lower than under π_6.

Appendix B.3 Proof of Lemma 1

The proof is immediate for policies π_1 and π_6. Estevan et al. (2018) analyzed policy π_1 and showed that $a_3(\pi_1, \sigma^*) \geq a_2(\pi_1, \sigma^*) \geq a_1(\pi_1, \sigma^*)$ holds for the unique equilibrium σ^*. Under policy π_6, there is no relocation and $a_i^N(\pi_6)$ is determined by $F_i(a_i^N(\pi_6)) = 1 - n$ for each $i = 1, 2, 3$. Therefore, by the FOSD of F_i's, $a_3^N(\pi_6) \geq a_2^N(\pi_6) \geq a_1^N(\pi_6)$ holds, implying $a_3(\pi_6) \geq a_2(\pi_6) \geq a_1(\pi_6)$.

Consider policy π_3 and take any equilibrium σ. First, we can show that $a_{i+1}(\pi_3, \sigma) = a^L(\pi_3, \sigma)$ holds when $a_i(\pi_3, \sigma) = a^L(\pi_3, \sigma)$ for $i = 1, 2$. Suppose $a_i(\pi_3, \sigma) = a^L(\pi_3, \sigma)$. $\bar{g}$ students at high school s_i cannot be admitted to U if their achievement a is lower than $\underline{a}(\pi_3, \sigma)$, and $\underline{g}$ students at high school s_i are not when $a < a^L(\pi_3, \sigma)$. Since $a_i(\pi_3, \sigma) = a^L(\pi_3, \sigma)$ is weakly higher than the cutoff scores of all other schools, there are no students migrating to s_i. Then,

$$\frac{1 - F_i(a^L(\pi_3, \sigma))}{1 - F_i(a^L(\pi_3, \sigma)) + p_i^{\bar{g}} F_i(\underline{a}(\pi_3, \sigma)) + p_i^{\underline{g}} F_i(a^L(\pi_3, \sigma))} \geq n,$$

which is equivalent to $1 - F_i(a^L(\pi_3, \sigma)) \geq \frac{n}{1-n}[p_i^{\bar{g}} F_i(\underline{a}(\pi_3, \sigma)) + p_i^{\underline{g}} F_i(a^L(\pi_3, \sigma))]$. By the FOSD of F_i's, $p_i^{\bar{g}} \leq p_{i+1}^{\bar{g}}$ and $a^L(\pi_3, \sigma) \geq \underline{a}(\pi_3, \sigma)$,

$$\begin{aligned}
&1 - F_{i+1}(a^L(\pi_3, \sigma)) \geq 1 - F_i(a^L(\pi_3, \sigma)) \\
\geq &\frac{n}{1-n}[p_i^{\bar{g}} F_i(\underline{a}(\pi_3, \sigma)) + p_i^{\underline{g}} F_i(a^L(\pi_3, \sigma))] \geq \frac{n}{1-n}[p_{i+1}^{\bar{g}} F_i(\underline{a}(\pi_3, \sigma)) + p_{i+1}^{\underline{g}} F_i(a^L(\pi_3, \sigma))] \\
\geq &\frac{n}{1-n}[p_{i+1}^{\bar{g}} F_{i+1}(\underline{a}(\pi_3, \sigma)) + p_{i+1}^{\underline{g}} F_{i+1}(a^L(\pi_3, \sigma))].
\end{aligned}$$

If $a^L(\pi_3, \sigma) > a_{i+1}(\pi_3, \sigma)$ happens, we would have

$$\begin{aligned}
n = &\frac{1 - F_{i+1}(a_{i+1}(\pi_3, \sigma)) + M_{i+1}}{1 - F_{i+1}(a_{i+1}(\pi_3, \sigma)) + M_{i+1} + p_{i+1}^{\bar{g}} F_{i+1}(\underline{a}(\pi_3, \sigma)) + p_{i+1}^{\underline{g}} F_{i+1}(a_{i+1}(\pi_3, \sigma))} \\
> &\frac{1 - F_{i+1}(a^L(\pi_3, \sigma))}{1 - F_{i+1}(a^L(\pi_3, \sigma)) + p_{i+1}^{\bar{g}} F_{i+1}(\underline{a}(\pi_3, \sigma)) + p_{i+1}^{\underline{g}} F_{i+1}(a^L(\pi_3, \sigma))} \geq n,
\end{aligned}$$

where $M_{i+1} \geq 0$ is the population of students who move to s_{i+1}. This is a contradiction, and $a_{i+1}(\pi_3, \sigma) = a^L(\pi_3, \sigma)$ must hold.

Second, we show that when only students from s_3 are admitted at large, $(a_3(\pi_3, \sigma) \geq)$ $a_2(\pi_3, \sigma) \geq a_1(\pi_3, \sigma)$ holds. Since $a_3(\pi_3, \sigma)$ is the highest cutoff score in this case, toward a contradiction, suppose $a_3(\pi_3, \sigma) \geq a_1(\pi_3, \sigma) > a_2(\pi_3, \sigma)$. Then, we have $a^L(\pi_3, \sigma) > a_2^N(\pi_3, \sigma)$. By $a_1(\pi_3, \sigma) > a_2(\pi_3, \sigma)$, no student moves to s_1 since moving to s_2 is less costly for students at s_3. $\bar{g}$ students at s_1 are not admitted to U if $a < a_2^N(\pi_3, \sigma)$, and $\underline{g}$ students at s_1 are not if $a < a_1(\pi_3, \sigma)$. Students at s_2 are not admitted to U if $a < a_2^N(\pi_3, \sigma)$. Let $M_2 \geq 0$ be the population of students who move to s_2. Then, we have

$$n = \frac{q_2[1 - F_2(a_2^N(\pi_3, \sigma))] + M_2}{q_2[1 - F_2(a_2^N(\pi_3, \sigma))] + M_2 + q_2 F_2(a_2^N(\pi_3, \sigma))} = \frac{1 - F_1(a_1^N(\pi_3, \sigma))}{1 - F_1(a_1^N(\pi_3, \sigma)) + p_1^{\bar{g}} F_1(a_2^N(\pi_3, \sigma)) + p_1^{\underline{g}} F_1(a_1(\pi_3, \sigma))},$$

which is equivalent to

$$\frac{F_2(a_2^N(\pi_3, \sigma))}{p_1^{\bar{g}} F_1(a_2^N(\pi_3, \sigma)) + p_1^{\underline{g}} F_1(a_1(\pi_3, \sigma))} = \frac{1 - F_2(a_2^N(\pi_3, \sigma)) + M_2/q_2}{1 - F_1(a_1^N(\pi_3, \sigma))}.$$

Since F_2 strictly first-order stochastically dominates F_1 and $a_1^N(\pi_3, \sigma) > a_2^N(\pi_3, \sigma)$,

$$\begin{aligned} 1 &> \frac{F_1(a_2^N(\pi_3, \sigma))}{p_1^{\bar{g}} F_1(a_2^N(\pi_3, \sigma)) + p_1^{\underline{g}} F_1(a_1(\pi_3, \sigma))} > \frac{F_2(a_2^N(\pi_3, \sigma))}{p_1^{\bar{g}} F_1(a_2^N(\pi_3, \sigma)) + p_1^{\underline{g}} F_1(a_1(\pi_3, \sigma))} \\ &= \frac{1 - F_2(a_2^N(\pi_3, \sigma)) + M_2/q_2}{1 - F_1(a_1^N(\pi_3, \sigma))} \geq \frac{1 - F_2(a_2^N(\pi_3, \sigma))}{1 - F_1(a_1^N(\pi_3, \sigma))} \geq \frac{1 - F_2(a_1^N(\pi_3, \sigma))}{1 - F_1(a_1^N(\pi_3, \sigma))}. \end{aligned}$$

But this contradicts the fact that F_2 strictly first-order stochastically dominates F_1.

Appendix B.4 Proof of Lemma 2

The proof consists of the following two parts.

[1] $a_2(\pi, \sigma) \geq a_1(\pi, \sigma)$ for any equilibrium σ of policy $\pi \in \{\pi_2, \pi_4, \pi_5\}$

Toward a contradiction, suppose $a_1(\pi, \sigma) > a_2(\pi, \sigma)$. For this to hold, we must have $a_1^N(\pi, \sigma) > a_2^N(\pi, \sigma)$ and $a^L(\pi, \sigma) > a_2^N(\pi, \sigma)$. Note that by $a_1(\pi, \sigma) > a_2(\pi, \sigma)$, no student moves to s_1 since moving to s_2 is less costly for students at s_3. Also, by $a^L(\pi, \sigma) > a_2^N(\pi, \sigma)$, no students from s_2 are admitted at large.

Consider policy π_2. $\bar{g}$ students at s_1 are not admitted to U if $a < \underline{a}_{2,3}(\pi_2, \sigma)$, and $\underline{g}$ students at s_1 are not admitted to U if $a < a_2^N(\pi_2, \sigma)$. Let $L_1 \geq 0$ be the population of s_1 students who are admitted at large. Students at s_2 are not admitted to U if $a < \underline{a}_{2,3}(\pi_2, \sigma)$. Let $M_2 \geq 0$ be the population of students who move to s_2. Then, we have

$$n = \frac{q_2[1 - F_2(a_2^N(\pi_2, \sigma))] + M_2}{q_2[1 - F_2(a_2^N(\pi_2, \sigma))] + M_2 + q_2 F_2(\underline{a}_{2,3}(\pi_2, \sigma))} = \frac{1 - F_1(a_1^N(\pi_2, \sigma))}{1 - F_1(a_1^N(\pi_2, \sigma)) + L_1 + p_1^{\bar{g}} F_1(\underline{a}_{2,3}(\pi_2, \sigma)) + p_1^{\underline{g}} F_1(a_2^N(\pi_2, \sigma))},$$

which is equivalent to

$$\frac{F_2(\underline{a}_{2,3}(\pi_2, \sigma))}{L_1 + p_1^{\bar{g}} F_1(\underline{a}_{2,3}(\pi_2, \sigma)) + p_1^{\underline{g}} F_1(a_2^N(\pi_2, \sigma))} = \frac{1 - F_2(a_2^N(\pi_2, \sigma)) + M_2/q_2}{1 - F_1(a_1^N(\pi_2, \sigma))}.$$

Since F_2 strictly first-order stochastically dominates F_1 and $a_1^N(\pi_2, \sigma) > a_2^N(\pi_2, \sigma) \geq \underline{a}_{2,3}(\pi_2, \sigma)$,

$$\begin{aligned} 1 &\geq \frac{F_1(\underline{a}_{2,3}(\pi_2, \sigma))}{L_1 + p_1^{\bar{g}} F_1(\underline{a}_{2,3}(\pi_2, \sigma)) + p_1^{\underline{g}} F_1(a_2^N(\pi_2, \sigma))} > \frac{F_2(\underline{a}_{2,3}(\pi_2, \sigma))}{L_1 + p_1^{\bar{g}} F_1(\underline{a}_{2,3}(\pi_2, \sigma)) + p_1^{\underline{g}} F_1(a_2^N(\pi_2, \sigma))} \\ &= \frac{1 - F_2(a_2^N(\pi_2, \sigma)) + M_2/q_2}{1 - F_1(a_1^N(\pi_2, \sigma))} \geq \frac{1 - F_2(a_2^N(\pi_2, \sigma))}{1 - F_1(a_1^N(\pi_2, \sigma))} \geq \frac{1 - F_2(a_1^N(\pi_2, \sigma))}{1 - F_1(a_1^N(\pi_2, \sigma))}. \end{aligned}$$

But this contradicts the fact that F_2 strictly first-order stochastically dominates F_1.

Consider policy π_4. Students at s_1 are not admitted to U if $a < a_2^N(\pi_4, \sigma)$. Let $L_1 \geq 0$ be the population of s_1 students who are admitted at large. Students at s_2 are not admitted to U if $a < \underline{a}_{2,3}(\pi_4, \sigma)$. Let $M_2 \geq 0$ be the population of students who move to s_2. Then, we have

$$n = \frac{q_2[1 - F_2(a_2^N(\pi_4, \sigma))] + M_2}{q_2[1 - F_2(a_2^N(\pi_4, \sigma))] + M_2 + q_2 F_2(\underline{a}_{2,3}(\pi_4, \sigma))} = \frac{1 - F_1(a_1^N(\pi_4, \sigma))}{1 - F_1(a_1^N(\pi_4, \sigma)) + L_1 + F_1(a_2^N(\pi_4, \sigma))},$$

which is equivalent to

$$\frac{F_2(\underline{a}_{2,3}(\pi_4, \sigma))}{L_1 + F_1(a_2^N(\pi_4, \sigma))} = \frac{1 - F_2(a_2^N(\pi_4, \sigma)) + M_2/q_2}{1 - F_1(a_1^N(\pi_4, \sigma))}.$$

Since F_2 strictly first-order stochastically dominates F_1 and $a_1^N(\pi_4, \sigma) > a_2^N(\pi_4, \sigma) \geq \underline{a}_{2,3}(\pi_4, \sigma)$,

$$\begin{aligned} 1 \quad &\geq \quad \frac{F_2(a_2^N(\pi_4, \sigma))}{L_1 + F_1(a_2^N(\pi_4, \sigma))} \geq \frac{F_2(\underline{a}_{2,3}(\pi_4, \sigma))}{L_1 + F_1(a_2^N(\pi_4, \sigma))} \\ &= \quad \frac{1 - F_2(a_2^N(\pi_4, \sigma)) + M_2/q_2}{1 - F_1(a_1^N(\pi_4, \sigma))} \geq \frac{1 - F_2(a_2^N(\pi_4, \sigma))}{1 - F_1(a_1^N(\pi_4, \sigma))} \geq \frac{1 - F_2(a_1^N(\pi_4, \sigma))}{1 - F_1(a_1^N(\pi_4, \sigma))}. \end{aligned}$$

But this contradicts the fact that F_2 strictly first-order stochastically dominates F_1.

Consider policy π_5. $\bar{g}$ students at s_1 are not admitted to U if $a < a_2^N(\pi_5, \sigma)$, and $\underline{g}$ students at s_1 are not admitted to U if $a < a_1(\pi_5, \sigma)$. Let $L_1 \geq 0$ be the population of s_1 students who are admitted at large. $\bar{g}$ students at s_2 are not admitted to U if $a < \underline{a}_{2,3}(\pi_5, \sigma)$, and $\underline{g}$ students at s_2 are not admitted to U if $a < a_2^N(\pi_5, \sigma)$. Let $M_2 \geq 0$ be the population of students who move to s_2. Then, we have

$$\begin{aligned} n &= \frac{q_2[1 - F_2(a_2^N(\pi_5, \sigma))] + M_2}{q_2[1 - F_2(a_2^N(\pi_5, \sigma))] + M_2 + p_2^{\bar{g}} q_2 F_2(\underline{a}_{2,3}(\pi_5, \sigma)) + p_2^{\underline{g}} q_2 F_2(a_2^N(\pi_5, \sigma))} \\ &= \frac{1 - F_1(a_1^N(\pi_5, \sigma))}{1 - F_1(a_1^N(\pi_5, \sigma)) + L_1 + p_1^{\bar{g}} F_1(a_2^N(\pi_5, \sigma)) + p_1^{\underline{g}} F_1(a_1(\pi_5, \sigma))}, \end{aligned}$$

which is equivalent to

$$\frac{p_2^{\bar{g}} F_2(\underline{a}_{2,3}(\pi_5, \sigma)) + p_2^{\underline{g}} F_2(a_2^N(\pi_5, \sigma))}{L_1 + p_1^{\bar{g}} F_1(a_2^N(\pi_5, \sigma)) + p_1^{\underline{g}} F_1(a_1(\pi_5, \sigma))} = \frac{1 - F_2(a_2^N(\pi_5, \sigma)) + M_2/q_2}{1 - F_1(a_1^N(\pi_5, \sigma))}.$$

Since F_2 strictly first-order stochastically dominates F_1 and $a_1^N(\pi_5, \sigma) > a_2^N(\pi_5, \sigma) \geq \underline{a}_{2,3}(\pi_5, \sigma)$,

$$\begin{aligned} 1 \quad &> \quad \frac{F_2(a_2^N(\pi_5, \sigma))}{L_1 + p_1^{\bar{g}} F_1(a_2^N(\pi_5, \sigma)) + p_1^{\underline{g}} F_1(a_1(\pi_5, \sigma))} \geq \frac{p_2^{\bar{g}} F_2(\underline{a}_{2,3}(\pi_5, \sigma)) + p_2^{\underline{g}} F_2(a_2^N(\pi_5, \sigma))}{L_1 + p_1^{\bar{g}} F_1(a_2^N(\pi_5, \sigma)) + p_1^{\underline{g}} F_1(a_1(\pi_5, \sigma))} \\ &= \quad \frac{1 - F_2(a_2^N(\pi_5, \sigma)) + M_2/q_2}{1 - F_1(a_1^N(\pi_5, \sigma))} \geq \frac{1 - F_2(a_2^N(\pi_5, \sigma))}{1 - F_1(a_1^N(\pi_5, \sigma))} \geq \frac{1 - F_2(a_1^N(\pi_5, \sigma))}{1 - F_1(a_1^N(\pi_5, \sigma))}. \end{aligned}$$

But this contradicts the fact that F_2 strictly first-order stochastically dominates F_1.

[2] $a_3(\pi,\sigma) \geq a_1(\pi,\sigma)$ for any equilibrium σ of policy $\pi \in \{\pi_2, \pi_4, \pi_5\}$

Toward a contradiction, suppose $a_1(\pi,\sigma) > a_3(\pi,\sigma)$. By [1], we must have $a_2(\pi,\sigma) \geq a_1(\pi,\sigma) > a_3(\pi,\sigma)$. This implies $a^L(\pi,\sigma) > a_3^N(\pi,\sigma)$ and $a_2^N(\pi,\sigma) > a_3^N(\pi,\sigma)$. Then, no students move to s_2, and no students at s_3 are admitted to U at large.

Consider policy $\pi \in \{\pi_2, \pi_4\}$. Students at s_2 are not admitted to U if $a < a_3^N(\pi,\sigma)$. Let $L_2 \geq 0$ be the population of s_2 students who are admitted at large. Students at s_3 are not admitted to U if $a < a_3^N(\pi,\sigma)$. Let $M_3 \geq 0$ be the population of students who move to s_3. Then, we have

$$n = \frac{q_3[1-F_3(a_3^N(\pi,\sigma))] + M_3}{q_3[1-F_3(a_3^N(\pi,\sigma))] + M_3 + q_3F_3(a_3^N(\pi,\sigma))} = \frac{1-F_2(a_2^N(\pi,\sigma))}{1-F_2(a_2^N(\pi,\sigma)) + L_2 + F_2(a_3^N(\pi,\sigma))}$$

which is equivalent to

$$\frac{F_3(a_3^N(\pi,\sigma))}{L_2 + F_2(a_3^N(\pi,\sigma))} = \frac{1-F_3(a_3^N(\pi,\sigma)) + M_3/q_3}{1-F_2(a_2^N(\pi,\sigma))}.$$

Since F_3 strictly first-order stochastically dominates F_2 and $a_2^N(\pi,\sigma) > a_3^N(\pi,\sigma)$,

$$1 > \frac{F_3(a_3^N(\pi,\sigma))}{L_2 + F_2(a_3^N(\pi,\sigma))} = \frac{1-F_3(a_3^N(\pi,\sigma)) + M_3/q_3}{1-F_2(a_2^N(\pi,\sigma))} \geq \frac{1-F_3(a_3^N(\pi,\sigma))}{1-F_2(a_2^N(\pi,\sigma))} \geq \frac{1-F_3(a_2^N(\pi,\sigma))}{1-F_2(a_2^N(\pi,\sigma))}.$$

But this contradicts the fact that F_3 strictly first-order stochastically dominates F_2.

Consider policy π_5. $\bar{g}$ students at s_2 are not admitted to U if $a < a_3^N(\pi_5,\sigma)$, and $\underline{g}$ students at s_2 are not admitted to U if $a < a_2(\pi_5,\sigma)$. Let $L_2 \geq 0$ be the population of s_2 students who are admitted at large. Students at s_3 are not admitted to U if $a < a_3^N(\pi_5,\sigma)$. Let $M_3 \geq 0$ be the population of students who move to s_3. Then, we have

$$n = \frac{q_3[1-F_3(a_3^N(\pi_5,\sigma))] + M_3}{q_3[1-F_3(a_3^N(\pi_5,\sigma))] + M_3 + q_3F_3(a_3^N(\pi_5,\sigma))} = \frac{1-F_2(a_2^N(\pi_5,\sigma))}{1-F_2(a_2^N(\pi_5,\sigma)) + L_2 + p_2^{\bar{g}}F_2(a_3^N(\pi_5,\sigma)) + p_2^{\underline{g}}F_2(a_2(\pi_5,\sigma))},$$

which is equivalent to

$$\frac{F_3(a_3^N(\pi_5,\sigma))}{L_2 + p_2^{\bar{g}}F_2(a_3^N(\pi_5,\sigma)) + p_2^{\underline{g}}F_2(a_2(\pi_5,\sigma))} = \frac{1-F_3(a_3^N(\pi_5,\sigma)) + M_3/q_3}{1-F_2(a_2^N(\pi_5,\sigma))}.$$

Since F_3 strictly first-order stochastically dominates F_2 and $a_2^N(\pi_5,\sigma) > a_3^N(\pi_5,\sigma)$,

$$\begin{aligned} 1 \ &\geq \ \frac{F_2(a_3^N(\pi_5,\sigma))}{L_2 + p_2^{\bar{g}}F_2(a_3^N(\pi_5,\sigma)) + p_2^{\underline{g}}F_2(a_2(\pi_5,\sigma))} > \frac{F_3(a_3^N(\pi_5,\sigma))}{L_2 + p_2^{\bar{g}}F_2(a_3^N(\pi_5,\sigma)) + p_2^{\underline{g}}F_2(a_2(\pi_5,\sigma))} \\ &= \ \frac{1-F_3(a_3^N(\pi_5,\sigma)) + M_3/q_3}{1-F_2(a_2^N(\pi_5,\sigma))} \geq \frac{1-F_3(a_3^N(\pi_5,\sigma))}{1-F_2(a_2^N(\pi_5,\sigma))} \geq \frac{1-F_3(a_2^N(\pi_5,\sigma))}{1-F_2(a_2^N(\pi_5,\sigma))}. \end{aligned}$$

But this contradicts the fact that F_3 strictly first-order stochastically dominates F_2.

Appendix B.5 Proof of Proposition 3

Let $m_{i,j}(\pi, \sigma)$ be the population of students who move from s_i to s_j in the equilibrium σ of policy π.

Proposition 1 implies that policy π_1 achieves weakly higher integration than π_2 and π_3. Proposition 2 shows that π_6 achieves weakly higher integration than π_1. Therefore, to show the optimality of π_6 when π_4 is not feasible, it suffices to prove that π_6 achieves weakly higher integration than π_5.

Given that π_6 is biased toward the type-$\bar{g}$ students, it is equivalent to showing that the share of $\underline{g}$ students at U under π_5 is lower than under π_6 in any equilibrium. Under policy $\pi_k \in \{\pi_5, \pi_6\}$, the population of $\underline{g}$ students admitted to U given cutoff scores (a_1, a_2, a_3) is

$$p_3^{\underline{g}} q_3[1 - F_3(a_3)] + p_2^{\underline{g}} q_2[1 - F_2(a_2)] + p_1^{\underline{g}} q_1[1 - F_1(a_1)].$$

Then, it suffices to show $a_i(\pi_5, \sigma) \geq a_i(\pi_6)$ for each $i \in \{1, 2, 3\}$ for any equilibrium σ of π_5.

Consider when $a_3(\pi_6) = a_2(\pi_6) = a_1(\pi_6)$. We can show $a_3(\pi_5, \sigma) = a_2(\pi_5, \sigma) = a_1(\pi_5, \sigma)$ for any equilibrium σ of π_5, implying $a_i(\pi_5, \sigma) = a_i(\pi_6)$ for each $i \in \{1, 2, 3\}$ by the market-clearing conditions. Suppose that $a_3(\pi_5, \sigma) = a_2(\pi_5, \sigma) = a_1(\pi_5, \sigma)$ does not hold for some equilibrium σ of π_5. By Lemma 2, $a^L(\pi_5, \sigma) > a_1^N(\pi_5, \sigma)$ must be the case and no students from s_1 are admitted at large. Since F_2 first-order stochastically dominates F_1, $a_3^N(\pi_5, \sigma) \geq a^L(\pi_5, \sigma) > a_2^N(\pi_5, \sigma) = a_1^N(\pi_5, \sigma)$ cannot happen and we must have $a_2^N(\pi_5, \sigma) > a_1^N(\pi_5, \sigma)$. Then for s_1,

$$q_1[1 - F_1(a_1(\pi_5, \sigma))] + m_{2,1}(\pi_5, \sigma) = n[q_1 + m_{2,1}(\pi_5, \sigma)],$$

where $m_{2,1}(\pi_5, \sigma) > 0$. $q_1[1 - F_1(a_1(\pi_5, \sigma))] = nq_1 - (1 - n)m_{2,1}(\pi_5, \sigma) < nq_1 \leq q_1[1 - F_1(a_1(\pi_6))]$ implies $a_1(\pi_5, \sigma) > a_1(\pi_6)$, and hence $a_2(\pi_5, \sigma) > a_2(\pi_6)$ and $a_3(\pi_5, \sigma) > a_3(\pi_6)$ also hold. But this is a contradiction because the total population of students admitted to U under π_5 would be strictly smaller than that under π_6.

Consider when $a_3(\pi_6) = a_2(\pi_6) > a_1(\pi_6)$. In this case, $a_3(\pi_6) = a_2(\pi_6) = a^L(\pi_6)$ holds and the market-clearing conditions are

$$q_3[1 - F_3(a^L(\pi_6))] + q_2[1 - F_2(a^L(\pi_6))] = n[q_3 + q_2] + (q_U - n)$$
$$q_1[1 - F_1(a_1(\pi_6))] = nq_1.$$

Note that $a_3(\pi_5, \sigma) = a_2(\pi_5, \sigma) = a_1(\pi_5, \sigma)$ cannot happen in this case. First, suppose $a_3(\pi_5, \sigma) = a_2(\pi_5, \sigma) = a^L(\pi_5, \sigma) > a_1(\pi_5, \sigma)$ holds in equilibrium σ. Then, we have

$$q_3[1 - F_3(a^L(\pi_5, \sigma))] + q_2[1 - F_2(a^L(\pi_5, \sigma))] = n[q_3 + q_2 - m_{2,1}(\pi_5, \sigma)] + (q_U - n)$$
$$q_1[1 - F_1(a_1(\pi_5, \sigma))] + m_{2,1}(\pi_5, \sigma) = n[q_1 + m_{2,1}(\pi_5, \sigma)].$$

By comparing these policies and by $m_{2,1}(\pi_5, \sigma) > 0$,

$$
\begin{aligned}
&q_3[1 - F_3(a^L(\pi_5, \sigma))] + q_2[1 - F_2(a^L(\pi_5, \sigma))] < q_3[1 - F_3(a^L(\pi_6))] + q_2[1 - F_2(a^L(\pi_6))] \\
&q_1[1 - F_1(a_1(\pi_5, \sigma))] < q_1[1 - F_1(a_1(\pi_6))],
\end{aligned}
$$

implying $a^L(\pi_5, \sigma) > a^L(\pi_6)$ and $a_1(\pi_5, \sigma) > a_1(\pi_6)$. Second, suppose that in equilibrium σ, there is only one $i \in \{2, 3\}$ such that $a_i(\pi_5, \sigma) = a^L(\pi_5, \sigma)$ and $a_i(\pi_5, \sigma) > a_j(\pi_5, \sigma)$ for $j \in \{2, 3\} \setminus \{i\}$. The market-clearing conditions for s_j under these two policies satisfy

$$
\begin{aligned}
&q_j[1 - F_j(a_j(\pi_6))] \geq nq_j \\
&q_j[1 - F_j(a_j(\pi_5, \sigma))] + m_{i,j}(\pi_5, \sigma) = n[q_j + m_{i,j}(\pi_5, \sigma) - m_{j,1}(\pi_5, \sigma)],
\end{aligned}
$$

where $m_{i,j}(\pi_5, \sigma) > 0$ and $m_{j,1}(\pi_5, \sigma) \geq 0$. Then, $q_j[1 - F_j(a_j(\pi_5, \sigma))] = nq_j - (1 - n)m_{i,j}(\pi_5, \sigma) - nm_{j,1}(\pi_5, \sigma) < q_j[1 - F_j(a_j(\pi_6))]$ implies $a_j(\pi_5, \sigma) > a_j(\pi_6)$, and we have $a_i(\pi_5, \sigma) > a_j(\pi_5, \sigma) > a_2(\pi_6) = a_3(\pi_6)$. The market-clearing condition for s_1 under (π_5, σ) is $q_1[1 - F_1(a_1(\pi_5, \sigma))] + m_{2,1}(\pi_5, \sigma) = n[q_1 + m_{2,1}(\pi_5, \sigma)]$, and we obtain $a_1(\pi_5, \sigma) \geq a_1(\pi_6)$ by $q_1[1 - F_1(a_1(\pi_5, \sigma))] \leq q_1[1 - F_1(a_1(\pi_6))]$.

Consider when $a_3(\pi_6) > a_2(\pi_6) > a_1(\pi_6)$. In this case, the market-clearing conditions are

$$
\begin{aligned}
q_3[1 - F_3(a_3(\pi_6))] &= nq_3 + (q_U - n) \\
q_2[1 - F_2(a_2(\pi_6))] &= nq_2 \\
q_1[1 - F_1(a_1(\pi_6))] &= nq_1.
\end{aligned}
$$

Note that $a_3(\pi_5, \sigma) = a_2(\pi_5, \sigma) = a_1(\pi_5, \sigma)$ cannot happen in this case. First, suppose that $a_3(\pi_5, \sigma) > a_2(\pi_5, \sigma) > a_1(\pi_5, \sigma)$ holds in equilibrium σ. Then, the market-clearing conditions under (π_5, σ) are

$$
\begin{aligned}
&q_3[1 - F_3(a_3(\pi_5, \sigma))] = n[q_3 - m_{3,2}(\pi_5, \sigma)] + (q_U - n) \\
&q_2[1 - F_2(a_2(\pi_5, \sigma))] + m_{3,2}(\pi_5, \sigma) = n[q_2 + m_{3,2}(\pi_5, \sigma) - m_{2,1}(\pi_5, \sigma)] \\
&q_1[1 - F_1(a_1(\pi_5, \sigma))] + m_{2,1}(\pi_5, \sigma) = n[q_1 + m_{2,1}(\pi_5, \sigma)].
\end{aligned}
$$

By $m_{2,1}(\pi_5, \sigma) > 0$ and $m_{3,2}(\pi_5, \sigma) > 0$, we have $q_i[1 - F_i(a_i(\pi_5, \sigma))] < q_i[1 - F_i(a_i(\pi_6))]$, implying $a_i(\pi_5, \sigma) > a_i(\pi_6)$, for each $i \in \{1, 2, 3\}$. Second, suppose that $a_2(\pi_5, \sigma) \geq a_3(\pi_5, \sigma) \geq a_1(\pi_5, \sigma)$ holds in equilibrium σ. Then for s_3, the market-clearing condition satisfies $q_3[1 - F_3(a_3(\pi_5, \sigma))] \leq nq_3 + (q_U - n) = q_3[1 - F_3(a_3(\pi_6))]$, implying $a_3(\pi_5, \sigma) \geq a_3(\pi_6)$. Therefore, we have $a_2(\pi_5, \sigma) \geq a_3(\pi_5, \sigma) \geq a_3(\pi_6) > a_2(\pi_6)$. The market-clearing condition for s_1 under (π_5, σ) is $q_1[1 - F_1(a_1(\pi_5, \sigma))] + m_{2,1}(\pi_5, \sigma) = n[q_1 + m_{2,1}(\pi_5, \sigma)]$, and we obtain $a_1(\pi_5, \sigma) \geq a_1(\pi_6)$ by $q_1[1 - F_1(a_1(\pi_5, \sigma))] \leq q_1[1 - F_1(a_1(\pi_6))]$.

Appendix B.6 Proof of Proposition 4

Let σ be an arbitrary equilibrium of policy π_4. By the same logic as the comparison of π_5 and π_6 in Proposition 3, $a_i(\pi_4, \sigma) \geq a_i(\pi_6)$ holds for any $i \in \{1, 2, 3\}$. Then, together with the assumption $\underline{a}_{2,3}(\pi_4, \sigma) \leq a_3(\pi_6)$, the population of $\bar{g}$ students at U under π_4 is weakly larger than under π_6:

$$\begin{aligned} & p_3^{\bar{g}} q_3[1 - F_3(\underline{a}_{2,3}(\pi_4, \sigma))] + p_2^{\bar{g}} q_2[1 - F_2(a_1(\pi_4, \sigma))] + p_1^{\bar{g}} q_1[1 - F_1(a_1(\pi_4, \sigma))] \\ \geq & p_3^{\bar{g}} q_3[1 - F_3(a_3(\pi_6))] + p_2^{\bar{g}} q_2[1 - F_2(a_2(\pi_6))] + p_1^{\bar{g}} q_1[1 - F_1(a_1(\pi_6))]. \end{aligned}$$

Since the share of $\bar{g}$ at U under π_6 is higher than the population average, the integration of U under (π_4, σ) is lower than under π_6.

Appendix B.7 Proof of Proposition 5

Let σ be an arbitrary equilibrium of policy π_4 such that $\underline{a}_{2,3}(\pi_4, \sigma) > a_3(\pi_6)$.

First, we can see that strictly less students from s_3 are admitted to U under (π_4, σ) than under π_6 because $q_3[1 - F_3(\underline{a}_{2,3}(\pi_4, \sigma))] < q_3[1 - F_3(a_3(\pi_6))]$. Next, we show that strictly less students from s_1 are admitted to U under (π_4, σ) than under π_6. Since $a_1(\pi_4, \sigma) = \min_i\{a_i(\pi_4, \sigma)\}$ and $a_3(\pi_6) = \max_i\{a_i(\pi_6)\}$, for the market-clearing conditions to hold under both policies, we must have $a_3(\pi_6) \geq a_1(\pi_4, \sigma)$. Then, the assumption $\underline{a}_{2,3}(\pi_4, \sigma) > a_3(\pi_6)$ implies $\underline{a}_{2,3}(\pi_4, \sigma) > a_1(\pi_4, \sigma)$. By $a_2(\pi_4, \sigma) > a_1(\pi_4, \sigma)$, $q_1 F_1(a_1(\pi_4, \sigma)) = (1 - n)[q_1 + m_{2,1}(\pi_4, \sigma)] > (1 - n)q_1$ holds. On the other hand, $F_1(a_1(\pi_6)) \leq 1 - n$ holds because of no move across high schools under π_6. Then, we have $a_1(\pi_4, \sigma) > a_1(\pi_6)$, implying $q_1[1 - F_1(\underline{a}_{1,2}(\pi_4, \sigma))] < q_1[1 - F_1(a_1(\pi_6))]$. By the two facts above, strictly more students from s_2 are admitted to U under (π_4, σ) than under π_6.

Given $p_3^{\bar{g}}$ and $p_1^{\bar{g}}$, let us consider two cases where $p_3^{\bar{g}} = p_2^{\bar{g}}$ and $p_2^{\bar{g}} = p_1^{\bar{g}}$. When $p_3^{\bar{g}} = p_2^{\bar{g}} > p_1^{\bar{g}}$, the admitted population of $\bar{g}$ students is strictly higher under (π_4, σ) than under π_6. Since the share of $\bar{g}$ students at U under π_6 is greater than the population average, the integration of U under (π_4, σ) is strictly lower than under π_6. When $p_3^{\bar{g}} > p_2^{\bar{g}} = p_1^{\bar{g}}$, the admitted population of $\bar{g}$ students is strictly lower under (π_4, σ) than under π_6. It is not clear which of (π_4, σ) and π_6 is optimal in this case because the share of $\bar{g}$ students at U may even be lower than the population average under (π_4, σ). But since the share of $\bar{g}$ students at U continuously decreases as $p_2^{\bar{g}}$ goes down and by $q_U > n$, there exists an interval $(x, y) \subset [p_1^{\bar{g}}, p_3^{\bar{g}}]$ such that for any $p_2^{\bar{g}} \in (x, y)$ the share of $\bar{g}$ students at U is between π_6 and the population average. In this interval, (π_4, σ) achieves a strictly higher integration than π_6.

Appendix B.8 Proof of Proposition 6

First, $q_U - n < \frac{2}{3}d_1 + \frac{1}{3}d_2$ implies that neither $\underline{a}_{2,3}(\pi_6) = a_1(\pi_6)$ nor $\underline{a}_{2,3}(\pi_4, \sigma) = a_1(\pi_4, \sigma)$ holds for any equilibrium σ of policy π_4. To see these, first, $\underline{a}_{2,3}(\pi_6) = a_1(\pi_6)$ would imply $a_3(\pi_6) = a_2(\pi_6) = a_1(\pi_6)$, which would require $q_U \geq \frac{1}{3}(3n + 2d_1 + d_2)$, but this is a contradiction. Next, if $\underline{a}_{2,3}(\pi_4, \sigma) = a_1(\pi_4, \sigma)$ holds, $a_2(\pi_4, \sigma) > a_3(\pi_4, \sigma) = a_1(\pi_4, \sigma)$ is impossible because no student from s_3 would be admitted at large and $a_3(\pi_4, \sigma) \geq 1 + d_1 + d_2 - n$ would hold but it is a contradiction to the fact that U admits $q_U > n$ students. Then, we would have $a_3(\pi_4, \sigma) \geq a_2(\pi_4, \sigma) = a_1(\pi_4, \sigma)$. But this would also imply $q_U \geq \frac{1}{3}(3n + 2d_1 + d_2)$ because a student from any high school with $a \geq a_1(\pi_4, \sigma)$ would be admitted to U and $a_1(\pi_4, \sigma) \leq 1 - n$. Then, it is again a contradiction to the condition $q_U - n < \frac{2}{3}d_1 + \frac{1}{3}d_2$. Therefore, we have both $\underline{a}_{2,3}(\pi_6) > a_1(\pi_6)$ and $\underline{a}_{2,3}(\pi_4, \sigma) > a_1(\pi_4, \sigma)$.

Consider cases with $a_3(\pi_6) = a_2(\pi_6) > a_1(\pi_6)$. By the same logic as the comparison between π_6 and π_5 in Proposition 3, $\underline{a}_{2,3}(\pi_4, \sigma) > a_3(\pi_6) = a_2(\pi_6)$ holds for any equilibrium σ of π_4 and the proof is done.

Next, consider cases with $a_3(\pi_6) > a_2(\pi_6) > a_1(\pi_6)$. Note that in this case, $a_3(\pi_6) = 1 + d_1 + d_2 - n - 3(q_U - n)$. First, suppose that any equilibrium σ of π_4 satisfies $a_2(\pi_4, \sigma) > a_3(\pi_4, \sigma) \geq a_1(\pi_4, \sigma)$. Consider an equilibrium $\hat{\sigma}$ in which all students in s_2 with achievement $a \in [a_3(\pi_4, \hat{\sigma}), a_2(\pi_4, \hat{\sigma}))$ relocate to s_1. Then, we have $a_2(\pi_4, \hat{\sigma}) > a_3(\pi_4, \hat{\sigma}) = 1 + d_1 + d_2 - n > 1 + d_1 + d_2 - n - 3(q_U - n) = a_3(\pi_6)$, implying $\underline{a}_{2,3}(\pi_4, \hat{\sigma}) > a_3(\pi_6)$. Second, suppose that there is an equilibrium $\bar{\sigma}$ of π_4 that satisfies $a_3(\pi_4, \bar{\sigma}) > a_2(\pi_4, \bar{\sigma}) \geq a_1(\pi_4, \bar{\sigma})$. The market-clearing conditions under π_4 imply

$$\frac{1}{3}[a_2(\pi_4, \bar{\sigma}) - d_1 - d_2 + a_1(\pi_4, \bar{\sigma}) - d_1] = (1 - n)\frac{1}{3}[1 - (a_2(\pi_4, \bar{\sigma}) - a_1(\pi_4, \bar{\sigma}))] - (q_U - n)$$
$$\frac{1}{3}a_1(\pi_4, \bar{\sigma}) = (1 - n)\frac{1}{3}[1 + (a_2(\pi_4, \bar{\sigma}) - a_1(\pi_4, \bar{\sigma}))]$$

By solving these, we obtain

$$a_2(\pi_4, \bar{\sigma}) = \frac{1}{4 - 3n}\Big[(2 - n)[3(1 - q_U) + 2d_1 + d_2] - 2(1 - n)\Big]$$
$$a_1(\pi_4, \bar{\sigma}) = \frac{1 - n}{4 - 3n}\Big[1 + 3(1 - q_U) + 2d_1 + d_2\Big]$$

Then, $q_U - n > -\frac{n}{6(1-n)}d_1 + \frac{1}{3}d_2$ implies

$$\begin{aligned} a_2(\pi_4, \bar{\sigma}) =& \frac{1}{4 - 3n}\Big[(2 - n)[3(1 - q_U) + 2d_1 + d_2] - 2(1 - n)\Big] \\ >& 1 + d_1 + d_2 - n - 3(q_U - n) = a_3(\pi_6). \end{aligned}$$